Get Ready for the Math Placement Test

MATH Placement Test Study Guide:
With Practice Questions and Solutions

The Quadratic Formula

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

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Get ready for the Math Placement Test

Dear students,

This workbook is one resource available to you as you begin to prepare for the Math Placement Test at the University of Baltimore.

The purpose of a placement test is to determine how much math you know and how well you know it. The placement test does not “grade” you. You cannot “pass” or “fail” this test! Instead, the placement test will measure your current skill set and place you into the appropriate level math class. You want to start your math classes in the course that is right for you. You do not need to cram or even study for the placement test. Instead, spend some time brushing up on the skills you already know, relax, and do your best.

A suggested path through this workbook:

1. Begin by taking the Math Placement Practice Test 1 found on page 9. Do not use a calculator.
2. Check your answers using the Answer Key found on the pages immediately after the test.
3. Review any concepts by:
   a. Finding the page in this workbook to review a short summary of the concept and to try more practice problems.
   b. Going online to free resources (like Khanacademy.org) for videos and practice problems.
   c. Meeting with a math coach or tutor.
   d. Checking out an Algebra Textbook from your Library.
4. Take the Math Placement Practice Test 2 found on page 123 and check your answers.
5. Review again, as needed.

Finally, get a good night’s rest and take your time while working on the test.

We wish you all the best!
1 Math Placement Practice Test 1

1. Which of the following represents three hundred two thousand, fifty-seven written in expanded form?
   A. $302 + 1,000 + 50 + 7$
   B. $300,000 + 2,000 + 50 + 7$
   C. $302,000 + 57$
   D. $300 + 2,000 + 50 + 7$

2. What is the word name for the number 3,605,499?
   A. three million, six hundred five thousand, four hundred ninety-nine
   B. 3 million, 605 thousand, 4 hundred ninety-nine
   C. three million, sixty five thousand, four hundred ninety-nine
   D. three billion, six hundred five thousand, four hundred ninety-nine

3. A marathon is a race that is 46,145 yards long. Round this number to the nearest ten.
   A. 46,150
   B. 46,140
   C. 46,000
   D. 46,100

4. Which expression correctly compares the numbers 1500 and 1348?
   A. $1500 < 1348$
   B. $1348 < 1500$
   C. $1500 = 1348$
   D. $1348 > 1500$

5. Shown below is a blueprint for a rectangular kennel at a pet hotel.

   ![Blueprint of a rectangular kennel](image)

   What is the total length of fencing needed to enclose the kennel? The total length needed is ____ feet.

   The solution is _________.

6. A DVD sales rack has 23 comedies, 13 dramas, 10 science fiction movies, and 31 action movies. How many DVDs are on the sales rack?
   A. 76
   B. 77
   C. 78
   D. 79

7. Subtract 842 − 465.

   The solution is _________.
8. Which of the following statements is true about the number $\frac{30}{2}$?
   A. It is contained in the set of rational numbers, but it is not contained in the set of natural numbers, whole number, integers, or irrational numbers.
   B. It is contained in the set of whole numbers and rational numbers, but it is not contained in the set of integers or irrational numbers.
   C. It is contained in the set of natural numbers, whole numbers, integers, and rational numbers, but it is not contained in the set of irrational numbers.
   D. It is contained in the set of irrational numbers and rational numbers, but it is not contained in the set of natural numbers or whole numbers.

9. Which list shows the set of integers?
   A. $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$
   B. $0, 1, 2, 3, \ldots$
   C. $\ldots, -3, -2, -1, 0$
   D. $1, 2, 3, \ldots$

10. Which expression has the greatest absolute value?
    A. $| -1 |$
    B. $| 5 |$
    C. $| -12 |$
    D. $| -8 |$

11. What is the opposite of 5?
    A. $-5$
    B. $-\frac{1}{5}$
    C. $\frac{1}{5}$
    D. 5

12. Which of the following numbers is shown plotted on the number line?
    A. 4.6
    B. $-\frac{15}{7}$
    C. $-2\frac{7}{8}$
    D. $-4\frac{3}{8}$
    E. $2\frac{7}{8}$

13. Which of the following statements is true?
    A. $-3.04 < -3.4$ because $-3.4$ is to the right of $-3.04$
    B. $-3.04 > -3.4$ because $-3.4$ is to the right of $-3.04$
    C. $-3.04 < -3.4$ because $-3.4$ is to the left of $-3.04$
    D. $-3.04 > -3.4$ because $-3.4$ is to the left of $-3.04$

14. Find $(-18) + (-15) + 10$
    A. $-23$
    B. $-43$
    C. 7
    D. $-13$
15. Add \((-71) + (-58)\)
   A. \(-13\)
   B. 13
   C. \(-129\)
   D. \(-149\)

16. Add \((-12) + 46\)
   A. 34
   B. \(-34\)
   C. 58
   D. \(-58\)

17. What is \(-4 + 0 + y, when y = 16?\)
   A. \(-20\)
   B. 12
   C. \(-12\)
   D. 0

18. Find \(-48a ÷ 8, when a = 1.\) Write the answer in simplest form.
   
   The solution is ____________.

19. Find \(-\frac{5}{6} ÷ (-\frac{1}{2})\). Write the answer in lowest terms.
   A. \(\frac{5}{3}\)
   B. \(-\frac{5}{3}\)
   C. \(\frac{5}{12}\)
   D. \(\frac{5}{5}\)

20. Three pairs of shoes cost $138. If each pair costs the same, how much did one pair cost?
   A. $46.00
   B. $49.00
   C. $135.00
   D. $141.00

21. Simplify: \(\frac{(3-3)^2}{3}\)
   A. 12
   B. 0
   C. 4
   D. \(-26\)
22. Match each expression on the left with an equivalent expression on the right. Some answer choices on the right will not be used.

A. \( \frac{2}{5} \)  
B. 0.75  
C. \( \frac{5}{8} \)  
D. 0.875  
E. 0.35  
F. \( \frac{3}{4} \)  
G. 0.625  
H. 0.4  
I. \( \frac{31}{40} \)  
J. \( \frac{7}{8} \)  
K. 0.58

23. Which of the following numbers is the largest?

6.392, 6.395, 6.3872, 6.3901

The solution is ___________.

24. Which of the following sentences is true? Use the number line to help you decide.

A. 6.2 is less than 5.7  
B. 5.6 is greater than 5.9  
C. 6.0 is greater than 5.9  
D. 6.1 is less than 5.8

25. Subtract 29.67 − 11.034

A. 18.33  
B. 18.573  
C. 18.636  
D. 18.033

26. Estimate 8,528 − 3,341 by first rounding to the nearest hundred.

A. 5,187  
B. 5,100  
C. 6,000  
D. 5,200

27. Add 21.25 + 7.06

A. 28.256  
B. 28.85  
C. 91.85  
D. 28.31

28. Which choice shows the product of 23 and 39?

A. 23 \times 39 = 677  
B. 23 + 39 = 62  
C. 23 \cdot 39 = 777  
D. (23)(39) = 897
29. At Frank’s work he can produce 4 widgets every hour. Which of the following expresses how many widgets Frank can make during an 8 hour day?
A. $8 + 8 + 8 + 8 + 8 + 8 + 8 + 8$
B. $(8)(4)$
C. $4 + 4 + 4 + 4$
D. $8 + 4$

30. Multiply $0.012(0.03)$
A. $0.00036$
B. $0.000036$
C. $0.36$
D. $0.036$

31. Divide $9.5085 ÷ 0.3$
A. $28.5255$
B. $31.695$
C. $3.1695$
D. $31.795$

32. Compute $\frac{9416}{312}$
A. $30 \text{ r} 56$
B. $31 \text{ r} 56$
C. $3 \text{ r} 56$
D. $30 \text{ r} 156$

33. Which of the following correctly shows the quotient of $55$ divided by $5$?
A. $5 ÷ 55 = 11$
B. $\frac{55}{5} = 11$
C. $\frac{55}{5} = 25$
D. $55 ÷ 5 = 50$

34. Which of the following equations is true?
A. $1 + 1 = 1 \cdot 1$
B. $6 \cdot 1 \cdot 0 = 6 + 1 + 0$
C. $7 \cdot 1 = 0 + 7$
D. $3 + 1 = 3 \cdot 0$

35. How could you correctly rewrite the equation $4(10 + 5) = 6(12 − 2)$ using the distributive property?
A. $40 + 30 = 72 − 2$
B. $20 + 40 = −12 + 72$
C. $40 + 5 = 72 − 2$
D. $40 + 20 = 72 − 12$

36. Which is equivalent to $2^4$?
A. $2 \cdot 4 = 8$
B. $2 \cdot 2 \cdot 2 \cdot 2 = 16$
C. $2 + 2 + 2 + 2 = 8$
D. $4 \cdot 4 = 16$
37. Write $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ in exponential notation.
   A. $4^6$
   B. $4^7$
   C. $7^4$
   D. 16,384

38. Match each expression on the left with an equivalent expression on the right. Some answer choices on the right will not be used.
   A. $\sqrt{4}$
   B. $\sqrt{100}$
   C. $\sqrt{49}$
   D. $\sqrt{64}$
   E. 2
   F. 7
   G. 8
   H. 9
   I. 10
   J. 16

39. Simplify $12 \div 3(4) + 2$
   The solution is __________.

40. A bowl holds 6 apples and 4 oranges. If Jasmine writes the fraction of fruit that are apples and does not reduce the fraction, which of the following would be the numerator of the fraction?
   A. $\frac{6}{10}$
   B. 6
   C. 4
   D. 10

41. Which of the following shows $\frac{18}{12}$ as a mixed number and $1\frac{7}{8}$ as an improper fraction?
   A. $\frac{18}{12} = \frac{3}{2}$ and $1\frac{7}{8} = \frac{15}{8}$
   B. $\frac{18}{12} = 1\frac{1}{2}$ and $1\frac{7}{8} = \frac{15}{8}$
   C. $\frac{18}{12} = 1\frac{6}{12}$ and $1\frac{7}{8} = \frac{15}{8}$
   D. $\frac{18}{12} = 1\frac{1}{2}$ and $1\frac{7}{8} = \frac{15}{8}$

42. Simplify $\frac{10}{16}$ to lowest terms and find an equivalent fraction that has a denominator of 32.
   A. $\frac{6}{16} \div \frac{20}{32}$
   B. $\frac{5}{10} \div \frac{8}{32}$
   C. $\frac{32}{16} \div \frac{20}{10}$
   D. $\frac{5}{10} \div \frac{20}{32}$
43. Find the equivalent fraction to \( \frac{28}{35} \) whose denominator is 10, 100 or 1000.

A. \( \frac{28}{100} \)

B. \( \frac{7}{10} \)

C. \( \frac{4}{10} \)

D. \( \frac{35}{100} \)

44. Which of the following is a true statement?

A. \( \frac{6}{16} > \frac{8}{24} \)

B. \( \frac{4}{8} < \frac{6}{16} \)

C. \( \frac{6}{16} > \frac{5}{10} \)

D. \( \frac{3}{5} < \frac{6}{16} \)

45. Multiply and simplify the answer: \( \frac{3}{5} \cdot \frac{5}{6} \)

A. \( \frac{15}{30} \)

B. \( \frac{1}{2} \)

C. \( 3 \)

D. \( \frac{18}{25} \)

46. Multiply and simplify the answer: \( 3 \frac{1}{2} \cdot 4 \frac{2}{5} \)

A. \( 15 \frac{2}{5} \)

B. \( 12 \frac{1}{5} \)

C. \( 15 \frac{9}{10} \)

D. \( 12 \frac{2}{5} \)

47. Divide and simplify the answer: \( 7 \frac{5}{6} \div \frac{2}{3} \)

A. \( 11 \frac{3}{4} \)

B. \( \frac{4}{47} \)

C. \( 2 \)

D. \( 4 \frac{1}{2} \)

48. Which fraction is equivalent to: \( \frac{5}{9} + \frac{1}{12} \)?

A. \( \frac{27}{24} \)

B. \( \frac{21}{27} \)

C. \( \frac{27}{23} \)

D. \( \frac{36}{29} \)

D. \( \frac{36}{36} \)
49. Peter mixes $4 \frac{1}{3}$ cups of orange juice, $2 \frac{1}{3}$ cups of ginger ale, and $6 \frac{1}{2}$ cups of strawberry lemonade to make some punch. What is the total number of cups of punch that Peter makes?
   A. $12 \frac{1}{2}$
   B. $12 \frac{3}{8}$
   C. $12 \frac{3}{5}$
   D. $13 \frac{1}{6}$

50. Subtract and simplify: $\frac{4}{15} - \frac{1}{6}$
   A. $\frac{3}{30}$
   B. $\frac{1}{10}$
   C. $\frac{3}{9}$
   D. $\frac{9}{90}$

51. Zo used $2 \frac{1}{2}$ cups of flour to bake a cake. She used $3 \frac{1}{4}$ cups of flour to bake a loaf of bread. How much more flour did Zo use to bake the loaf of bread than to bake the cake?
   A. $1 \frac{3}{4}$
   B. $\frac{3}{4}$
   C. $\frac{1}{4}$
   D. $5 \frac{3}{4}$

52. Rob planted 18 daffodil bulbs and 24 tulip bulbs in his garden last winter. If all the bulbs sprout and bloom this spring, what will be the ratio of tulips to daffodils in Rob’s garden? Express as a simplified ratio.
   A. $\frac{3}{4}$
   B. $\frac{3}{18}$
   C. $\frac{24}{24}$
   D. $\frac{24}{18}$

53. Tom traveled 354 miles in 6 hours. What is his unit rate of speed in miles per hour?
   The solution is ________ miles per hour.

54. Which of the following proportions is true?
   A. $\frac{12}{15} = \frac{21}{25}$
   B. $\frac{48}{72} = \frac{20}{40}$
   C. $\frac{25}{48} = \frac{90}{80}$
   D. $\frac{40}{40} = \frac{48}{48}$
55. Amy uses 3 cups of raisins for every 10 cups of trail mix she makes. How many cups of trail mix will she make if she uses 12 cups of raisins?

A. \(2 \frac{1}{5}\)
B. \(3 \frac{5}{5}\)
C. 10
D. 40

56. Which of the following shows the prime factorization of 36 using exponential notation?

A. \(2^2 \cdot 3^2\)
B. \(4 \cdot 9\)
C. \(2 \cdot 3^2\)
D. \(6^2\)

57. Determine whether 11 is prime, composite, or neither.

A. neither
B. prime
C. composite

58. What value of \(x\) makes the equation true? \(\frac{x}{5} + 3 = 1\)

A. –2
B. 2
C. 5
D. –10

59. Which equation is equivalent to \(4x + 3(2x - 4) = 2x\)

A. \(14x - 12 = 2x\)
B. \(14x + 28 = 2x\)
C. \(10x - 12 = 2x\)
D. \(10x + 12 = 2x\)

60. Find the base of a parallelogram with an area \((A)\) of 72 square inches and height \((h)\) of 4 inches. Use the formula for the area of a parallelogram, \(A = bh\).

A. \(\frac{1}{18}\)
B. 18
C. 68
D. 288

61. The solution to which inequality is graphed below?

A. \(12b > -4\)
B. \(6b > -18\)
C. \(−8b > 24\)
D. \(2b < -6\)
62. Which graph represents the solution to $7x > 21$ or $6x + 9 < 21$?

A. ![Graph A]
B. ![Graph B]
C. ![Graph C]
D. ![Graph D]

63. Solve for $k$: $-2 \leq -2k + 4 \leq 6$
   A. $-1 \geq k \geq -5$
   B. $3 \geq k \geq -1$
   C. $k \geq -1$
   D. $3 \leq k \leq -1$

64. In which quadrant is the point $(-2,3)$ located?
   A. Quadrant IV
   B. Quadrant I
   C. Quadrant II
   D. Quadrant III

65. Which point represents the ordered pair $(-3,-2)$?
   A. Point D
   B. Point B
   C. Point A
   D. Point C

66. Mia had $40. Then she started to receive $5 a week as an allowance. She plans to save all of her money for a bicycle and draws a graph of her planned savings. Mia lets $x$ represent the number of weeks she has received her allowance, and $y$ represent her total amount of money. Which of the following ordered pairs is on Mia’s graph?
   A. $(4,60)$
   B. $(1,40)$
   C. $(2,80)$
   D. $(6,30)$
67. Which of the following points lies on the line \( y = -x \)?
   A. (2,2)
   B. (−3, −3)
   C. (3,3)
   D. (2, −2)

68. What are the coordinates of the x-intercept of the line \( 3x - 2y = 12 \)?
   A. (0,6)
   B. (0,4)
   C. (−6,0)
   D. (4,0)

69. What is the slope of the line shown below?

   ![Graph](image)

   A. −2
   B. \(-\frac{1}{2}\)
   C. \(\frac{1}{2}\)
   D. 2

70. What is the slope of the line that contains the points (4,2) and (6,−2)?
   A. −2
   B. \(\frac{1}{4}\)
   C. 2
   D. \(-\frac{1}{2}\)

71. A line passes through (1, −1) and (3,5). What is the equation of the line in slope-intercept form?
   A. \(y = 3x - 4\)
   B. \(y = 3x - 5\)
   C. \(y = x - 2\)
   D. \(y = 2x - 3\)

72. What is the y-intercept of the line \( 2x - y = 4 \)?
   A. (0, −4)
   B. (0,2)
   C. (4,0)
   D. (2,0)

73. Write the equation of the line with a slope of 4 that contains the point (5,8).
   A. \(y = 4x + 8\)
   B. \(y = 4x + 28\)
   C. \(y = 4x - 12\)
   D. \(y = 4x - 27\)
74. Find the domain of the rational expression: \( \frac{6-x}{4x+20} \).
   A. All real numbers except -5
   B. All real numbers except -20
   C. All real numbers except 0
   D. All real numbers except 6

75. Simplify the expression: \( \frac{2x^2-11x-21}{2x^2+9x+9} \).
   A. \( \frac{-11x+21}{9x+9} \)
   B. \( \frac{-11x+7}{x+3} \)
   C. \( \frac{x-3}{x-7} \)
   D. \( \frac{x+7}{x+3} \)

76. Factor out the Greatest Common Factor (GCF): \( 64d^3 - 24d^2 \)
   A. \( 8d(8d^2 - 3d) \)
   B. \( 4d(16d^4 - 6d) \)
   C. \( 8d^2(8d - 3) \)
   D. \( 8d^2 \cdot 8d^3 - 8d^2 \cdot 3 \)

77. Factor by grouping. Then supply the term that is missing below.
   \( 4mn + 3m + 8n + 6 = (m + 2)(? + 3) \)
   The missing term is __________.

78. Factor \( x^2 + 9x + 20 \)
   A. \( (x + 2)(x + 7) \)
   B. \( (x + 6)(x + 3) \)
   C. \( (x + 2)(x + 10) \)
   D. \( (x + 4)(x + 5) \)

79. Factor \( 25x^2 - 36 \)
   A. \( (5x + 6)^2 \)
   B. \( (x - 4)(25x + 9) \)
   C. \( (5x - 6)^2 \)
   D. \( (5x - 6)(5x + 6) \)

80. Solve for \( x \): \( x^2 - 4x - 12 = 0 \)
   A. \( x = 0 \)
   B. \( x = 6 \)
   C. \( x = -2 \) or \( x = 6 \)
   D. \( x = 2 \) or \( x = -6 \)

81. Add and state the sum in simplest form: \( \frac{6}{5x} + \frac{7}{3y} \)
   A. \( \frac{30x+21y}{15xy} \)
   B. \( \frac{18y+35x}{15xy} \)
   C. \( \frac{5x+3y}{13} \)
   D. \( \frac{1}{15xy} \)
82. Solve the equation, eliminating any extraneous solutions: \[ \frac{x^2 + 4}{x-4} = \frac{20}{x-4} \]
A. \( x = 4 \)
B. \( x = -2, 2 \)
C. \( x = -4, 4 \)
D. \( x = -4 \)

83. Solve the equation: \[ \frac{x-5}{8} = \frac{x}{16} \]
A. \( x = 5 \)
B. \( x = -5 \)
C. \( x = -10 \)
D. \( x = 10 \)

84. The graph represents distance traveled varying directly with time.

[Image of a graph showing distance traveled over time]

What would be the distance traveled after 15 hours?
A. 750 miles
B. 600 miles
C. 675 miles
D. 40 miles

85. Which of the relations given by the following sets of ordered pairs is not a function?
A. \((-4, -4), (-3, -3), (0, 0), (3, -3), (4, -4)\)
B. \((5, 1), (4, 1), (3, 1), (2, 1), (1, 1)\)
C. \((-5, -7), (-4, -6), (-3, -5), (-2, -4), (-1, -3)\)
D. \((-4, -2), (-1, -1), (1, 3), (1, 4), (7, 10)\)

86. Which function below has the following domain and range?
Domain: \([-6, -5, 1, 2, 6]\)
Range: \([2, 3, 8]\)
A. \((2, 3), (-5, 2), (1, 8), (6, 3), (-6, 2)\)
B. \((2, -5), (8, 1), (3, 6), (2, -6), (3, 2)\)
C. \((-6, 6), (2, 8)\)
D. \((-6, 2), (-5, 3), (1, 8), (2, 5), (6, 9)\)
87. Which of the following is the graph of \( f(x) = -3x + 2 \)?

A. 

B. 

C. 

D. 

88. What are the domain and range of the real-valued function \( f(x) = \frac{2}{x} \)?

A. The domain is all real numbers except 0. The range is all real numbers.
B. The domain is all real numbers except 0. The range is all real numbers except 0.
C. The domain and the range are all real numbers.
D. The domain is \( x > 0 \). The range is \( f(x) > 0 \).

89. \( f(x) = 3x + 5 \)
\( g(x) = 4x^2 - 2 \)
\( h(x) = x^2 - 3x + 1 \)
Find \( f(x) - g(x) - h(x) \)
A. \( 5x^2 + 6x + 4 \)
B. \( -5x^2 + 6x + 6 \)
C. \( 5x^2 + 4 \)
D. \( -5x^2 + 6 \)
90. Between which two integers is the value of $\sqrt{77}$?
   A. 6 and 7  
   B. 7 and 8  
   C. 8 and 9  
   D. 9 and 10

91. Write $\sqrt{c^7}$ as an expression with a rational exponent.
   A. $c^\frac{7}{9}$  
   B. $c^7$  
   C. $c^{16}$  
   D. $c^{63}$

92. Write $6x^\frac{3}{5}$ in radical form.
   A. $6\sqrt[5]{x^3}$  
   B. $\sqrt[5]{6x^3}$  
   C. $6\sqrt[3]{x^5}$  
   D. $\sqrt[3]{6x^5}$

93. $\sqrt{16} + \sqrt{25} = ?$
   A. 4  
   B. 6  
   C. 9  
   D. 12

94. Perform the operations and simplify: $3\sqrt{12} - 5\sqrt{27} + 2\sqrt{75} = ?$
   A. $\sqrt[4]{51}$  
   B. $4\sqrt{3}$  
   C. $\sqrt{3}$  
   D. $6\sqrt{3} - 5$

95. Rationalize the denominator and simplify: $\frac{3+\sqrt{5}}{3\sqrt{5}}$
   A. $\frac{\sqrt{3b}+b^2}{3b}$  
   B. $\frac{3+b}{3b}$  
   C. $\frac{3\sqrt{b}+b}{3b}$  
   D. $\frac{\sqrt{b}}{b} + 1$

96. Which equation has $x = -2$ as a solution?
   A. $\sqrt{5} - 2x = 9$  
   B. $\sqrt{7} - x = 3$  
   C. $\sqrt{5x} - 1 = 3$  
   D. $\sqrt{2x} = 2$
97. Solve $x - 2 = \sqrt{x} + 18$
   A. 7
   B. no solution
   C. -2
   D. 7, -2

98. Solve $y^2 = \frac{1}{16}$
   A. $\pm 256$
   B. $\pm 8$
   C. $\pm 4$
   D. $\pm \frac{1}{4}$

99. Rewrite the equation in standard form and identify $a, b, and c$.
    $4x^2 + 2 = -8x - 7$
    A. $4x^2 + 8x + 9 = 0; a = 4, b = 8, c = 9$
    B. $4x^2 - 8x = 5; a = 4, b = -8, c = 5$
    C. $4x^2 + 8x = -9; a = 4, b = 8, c = -9$
    D. $4x^2 - 8x - 5 = 0; a = 4, b = -8, c = -5$

100. At what point do these two lines intersect?
    $y = 2x + 5$
    $y = -x - 4$
    A. (-1, -3)
    B. (3,1)
    C. (-3, -1)
    D. (1,3)
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2 Whole Numbers

2.1 Introduction to Whole Numbers

2.1.1 Rounding Whole Numbers (tens, hundreds, thousands)

Example

What is 45,671 rounded to the nearest thousand?

To round a number to a given place, you must find the digit in that place and look at the digit to its right.

To round 45,671 to the nearest thousand, first find the thousands place: 45,671

Then look at the digit in the hundreds place: 45,671

Because the digit is 6, and 6 > 5, you round up.

45,671 rounded to the nearest thousand is 46,000.

Practice problems

1) What is 12,603 rounded to the nearest thousand?

2) What is 77,803 rounded to the nearest hundred?

3) What is 17,863 rounded to the nearest ten?

Answers: 1) 13,000; 2) 77,800; 3) 17,860
2.1.2 Comparing Whole Numbers (Using > or <)

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*Example*

Which number is greater 451 or 453?

453 > 451 because 453 is 2 units to the right of 451.

*Practice problems*

1) Write in the correct inequality symbol between 72 and 720

2) Write in the correct inequality symbol between 13 and 10

Answers: 1) 72 < 720; 2) 13 > 10

2.2 Exponents and Square Roots

2.2.1 Evaluate an expression containing exponents and square roots

\[ 8^2 = 64 \text{ because } 8 \times 8 = 64 \]

\[ \sqrt{64} = 8 \text{ because } \sqrt{8^2} = 8 \]

*Example*

What is the value of \( 2^4 \)? The exponent tells you how many times to multiply the base. The base is 2 and the exponent is 4, so

\[ 2^4 = 2 \times 2 \times 2 \times 2 = 16 \]

*Practice problems*

1) What is the value of \( 3^2 \times 3^3 \)?

2) What is the value of \( \sqrt{121} \)?

Answers: 1) 243; 2) 11
2.2.2 Prime factorization of a whole number

Example

What are the prime factors of 40?
The prime factors of 40 are $2 \times 2 \times 2 \times 5$ or $2^3 \times 5$

Practice problems

1) What are the prime factors of 24?
2) What are the prime factors of 36?

Answers: 1) $2^3 \times 3$; 2) $2^2 \times 3^2$

2.3 Order of Operations

2.3.1 Simplify expressions

Example

Simplify:

10 $-$ (12 $-$ 2 $\times$ 2) + 11  Do the operations inside the parentheses first
10 $-$ (12 $-$ 4) + 11  Do multiplication before subtraction
10 $-$ 8 + 11  Complete the operations inside the parenthesis
2 + 11  Working left to right, complete the subtraction
13  Answer!
Practice problems

Simplify:
1) \( (5 \times 6 ÷ 2) + (14 ÷ 2) + 9 - 1 \)

2) \( (6 \times 2 ÷ 2)(5 \times 4) + 20 \)

Answers: 1) 30; 2) 140

3 Fractions and Mixed Numbers

3.1 Introduction to Fractions and Mixed Numbers

3.1.1 Convert improper fraction to mixed number

Example

Convert \( \frac{11}{4} \) to a mixed fraction

Divide \( 11 ÷ 4 = 2 \) with a remainder 3

\[ \frac{11}{4} = 2 \frac{3}{4} \]

Practice problems

1) Convert \( \frac{20}{3} \) to a mixed fraction

2) Convert \( \frac{35}{4} \) to a mixed fraction

Answers: 1) \( 6 \frac{2}{3} \); 2) \( 8 \frac{3}{4} \)
3.1.2 Convert mixed number to improper fraction

Example

Convert $3 \frac{2}{5}$ to an improper fraction

Multiply $3 \times 5 = 15$ and then add $15 + 2 = 17$

$3 \frac{2}{5} = \frac{17}{5}$

Practice problems

1) Convert $7 \frac{2}{3}$ to an improper fraction

2) Convert $6 \frac{1}{4}$ to an improper fraction

Answers: 1) $\frac{23}{3}$; 2) $\frac{25}{4}$

3.1.3 Simplify fraction to lowest terms

Example

Find a fraction equivalent to $\frac{4}{9}$ that has a denominator of 27.

To find an equivalent fraction, you can multiply the numerator and denominator by the same number. If you are starting with a denominator of 9, and ending with 27, you must multiply the numerator and denominator by 3.

So the answer is $\frac{4 \times 3}{9 \times 3} = \frac{12}{27}$
**Practice problems**

Simplify the fraction to the lowest terms:

1) \( \frac{8}{16} \)

2) \( \frac{7}{28} \)

Answers: 1) \( \frac{1}{2} \); 2) \( \frac{1}{4} \)

### 3.1.4 Comparing fractions

To compare two (or more) fractions, first find the Least Common Denominator (LCD) of the fractions. Second, rewrite each fraction as an equivalent fraction using the LCD. Third, compare the numerators.

**Example**

Order these fractions from least to greatest: \( \frac{3}{8}, \frac{5}{4}, \frac{1}{2} \)

The least common denominator is 8. So the above fractions can be written as:

\[
\frac{3}{8} \times \frac{2}{2} = \frac{6}{16}, \quad \frac{5}{4} \times \frac{2}{2} = \frac{10}{8}, \quad \frac{1}{2} \times \frac{4}{4} = \frac{4}{8}
\]

Since all fractions have the same denominator, we need to compare the numerator, therefore

\( \frac{3}{8}, \frac{10}{8}, \frac{4}{8} \) in order from least to greatest is \( \frac{3}{8}, \frac{4}{8}, \frac{10}{8} \)

Rewriting the fractions in simplest form: \( \frac{3}{8}, \frac{1}{2}, \frac{5}{4} \)

**Practice problems**

1) Order these fractions from least to greatest: \( \frac{7}{10}, \frac{3}{20}, \frac{7}{5} \)

2) Write in the correct inequality symbol between \( \frac{1}{3}, \frac{3}{4} \)

Answers: 1) \( \frac{3}{20}, \frac{7}{10}, \frac{7}{5} \); 2) \( \frac{1}{3} < \frac{3}{4} \)
3.2 Multiplying Fractions and Mixed Numbers

3.2.1 Multiply improper fractions

Steps:
1. Multiply the numerators
2. Multiply the denominators
3. Simplify, if necessary

Example

Multiply \( \frac{2}{3} \times \frac{7}{9} \)

To multiply fractions, you need to multiply the numerators and multiply the denominators:

\[
\frac{2}{3} \times \frac{7}{9} = \frac{2 \times 7}{3 \times 9} = \frac{14}{27}
\]

Practice problems

Multiply:
1) \( \frac{4}{7} \times \frac{2}{4} \)
2) \( \frac{10}{3} \times \frac{7}{72} \)

Answers: 1) \( \frac{2}{7} \), 2) \( \frac{35}{108} \)

3.2.2 Multiply mixed number fractions

Steps:
1. Convert mixed number fractions to improper fractions
2. Multiply the numerators
3. Multiply the denominators
4. Simplify, if necessary
5. Convert back to a mixed number

Example

Multiply \( 4 \frac{1}{2} \times 3 \frac{3}{4} \)

Convert each mixed fraction to an improper fraction, multiply, and simplify

\[
\frac{9}{2} \times \frac{15}{4} = \frac{9 \times 15}{2 \times 4} = \frac{135}{8} = 16 \frac{7}{8}
\]
Practice problems

Multiply:

1) \[ 10 \frac{1}{3} \times 5 \frac{4}{7} \]

2) \[ 3 \frac{1}{6} \times \frac{3}{4} \]

Answers: 1) \( \frac{57}{7} \); 2) \( \frac{3}{8} \)

3.3 Dividing Fractions and Mixed Numbers

3.3.1 Divide improper fractions

Example

Divide \( \frac{3}{5} \div \frac{6}{7} \)

\[
\frac{3}{5} \times \frac{7}{6} = \frac{3 \times 7}{5 \times 6} = \frac{21}{30} = \frac{7}{10}
\]

Practice problems

Divide:

1) \( \frac{4}{7} \div \frac{2}{4} \)

2) \( \frac{3}{2} \div \frac{6}{7} \)

Answers: 1) \( \frac{8}{7} \); 2) \( \frac{7}{4} \)
3.3.2 **Divide mixed number fractions**

**Dividing Mixed Numbers**

\[ \frac{3\frac{3}{4}}{2 + 1\frac{1}{4}} \]

**Step #1: Change to improper fractions**

\[ \frac{15 + 21}{4} = \frac{60}{84} \]

**Step #2: Multiply by the reciprocal**

\[ \frac{15}{4} \times \frac{4}{21} = \frac{60}{84} \]

**Step #3: Simplify if necessary**

\[ \frac{60 + 12}{84 + 12} = \frac{5}{7} \]

**Example**

Divide \( 4\frac{1}{2} \div 3\frac{3}{4} \)

\[ \frac{9}{2} \div \frac{15}{4} = \frac{9}{2} \times \frac{4}{15} = \frac{9 \times 4}{2 \times 15} = \frac{36}{30} = \frac{6}{5} = 1\frac{1}{5} \]

**Practice problems**

Divide:

1) \( \frac{1}{3} \div 5\frac{4}{7} \)

2) \( 3\frac{1}{6} \div \frac{3}{4} \)

Answers: 1) \( \frac{7}{117} \), 2) \( 4\frac{2}{9} \)

3.4 **Adding Fractions and Mixed Numbers**

3.4.1 **Add improper fractions**

**Adding Fractions**

**Adding Fractions with Like Denominators**

\[ \frac{1}{7} + \frac{3}{7} = \frac{1 + 3}{7} = \frac{4}{7} \]

Add the numerators, Denominator is unchanged.

**Adding Fractions with Unlike Denominators**

\[ \frac{1}{8} + \frac{2}{3} \]

Rewrite with common denominator

\[ 3 \times \frac{1}{8} + \frac{2}{3} \times 8 \]

\[ \frac{3 \times 1 + 2}{24} = \frac{3 + 16}{24} = \frac{19}{24} \]

Add the numerators

\[ \frac{3}{24} + \frac{16}{24} = \frac{19}{24} \]
**Example**

Add $\frac{2}{3} + \frac{1}{10}$

The least common multiple of 3 and 10 is 30, so this is the least common denominator. Rewrite both fractions with a denominator of 30:

$$\frac{20}{30} + \frac{3}{30}$$

Now you can add the numerators: $\frac{20}{30} + \frac{3}{30} = \frac{23}{30}$.

**Practice problems**

Add:

1) $\frac{2}{3} + \frac{1}{9}$

2) $\frac{1}{3} + \frac{4}{5}$

Answers: 1) $\frac{7}{9}$; 2) $\frac{17}{15}$

### 3.4.2 Add mixed number fractions

Convert each mixed fraction to an improper fraction, find the common denominator, and add.

**Example**

Add $4\frac{1}{2} + 3\frac{3}{4}$

First, convert to improper fractions. Second, rewrite the fractions so that they have the same denominator. The least common multiple of 2 and 4 is 4, so this is the least common denominator. Third, rewrite both fractions with a denominator of 4. Finally, add and simplify.

$$\frac{9}{2} + \frac{15}{4} = \frac{18}{4} + \frac{15}{4} = \frac{33}{4} = 8\frac{1}{4}$$

**Practice problems**

Add:

1) $10\frac{1}{3} + 5\frac{4}{7}$

2) $3\frac{1}{6} + \frac{3}{4}$

Answers: 1) $15\frac{19}{21}$; 2) $3\frac{11}{12}$
3.5 Subtracting Fractions and Mixed Numbers

3.5.1 Subtract improper fractions

*Example*

Subtract $\frac{2}{3} - \frac{1}{10}$

First, rewrite the fractions so that they have the same denominator. The least common multiple of 3 and 10 is 30, so this is the least common denominator. Second, rewrite both fractions with a denominator of 30:

$$\frac{20}{30} - \frac{3}{30}$$

Subtract the numerators: $\frac{20}{30} - \frac{3}{30} = \frac{17}{30}$

*Practice problems*

Subtract:

1) $\frac{2}{3} - \frac{1}{9}$

2) $\frac{1}{3} - \frac{1}{5}$

Answers: 1) $\frac{5}{9}$; 2) $\frac{2}{15}$

3.5.2 Subtract mixed number fractions

Convert each mixed fraction to an improper fraction, find the common denominator, and subtract.

*Example*

Subtract $4\frac{1}{2} - 3\frac{3}{4}$

First, convert to improper fractions. Second, rewrite the fractions so that they have the same denominator. The least common multiple of 2 and 4 is 4, so this is the least common denominator. Third, rewrite both fractions with a denominator of 4. Finally, subtract the numerators.

$$\frac{9}{2} - \frac{15}{4} = \frac{18}{4} - \frac{15}{4} = \frac{3}{4}$$
Practice problems

Subtract:

1) \(10 \frac{1}{3} - 5 \frac{4}{7}\)

2) \(3 \frac{1}{6} - \frac{3}{4}\)

Answers: 1) \(\frac{16}{21}\); 2) \(\frac{5}{12}\)

4 Decimals

4.1 Introduction to Decimals

4.1.1 Rounding Decimals (tenths, hundredths, thousandths)

<table>
<thead>
<tr>
<th>Place Value Chart (Decimals)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hundreds</td>
</tr>
<tr>
<td>100</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

Example

What is 0.631 rounded to the nearest tenth?

To round 0.631 to the nearest tenth, first find the tenths place: 0.\underline{6}31

Then look at the digit in the hundredths place: 0.\underline{63}1

Because the digit is 3, and 3 < 5, you keep the 6.

0.631 rounded to the nearest tenth is 0.6.

Practice problems

Round 1.1257 to the nearest:

1) tenth

2) hundredths

3) thousandths

Answers: 1) 1.1; 2) 1.13; 3) 1.126
### Example

Put the following numbers in order from least to greatest: 5.1, 5.15, 5.05, 5

First, stack the numbers and line up the decimal points. Write in the placeholder zeros:

- 5.100
- 5.151
- 5.050
- 5.000

Since all of numbers begin with 5, we only need to compare the numbers to the right of the decimal.

Ordering from least to greatest: 5.000, 5.050, 5.100, 5.151. Dropping the zero placeholders:

5, 5.05, 5.1, 5.151

### Practice problems

Place the appropriate symbol (=, <, or >) in between the numbers:

1) 13.7   13.1
2) 13.12  13.127
3) 13.1   13.100

Answers: 1) 13.7 > 13.1; 2) 13.12 < 13.127; 3) 13.1 = 13.100
4.2 Adding and Subtracting Decimals

4.2.1 Add Decimals

Example

Add: 3.25 + 0.041

You may add zeros to the right so each number has the same length. Add:

\[
\begin{array}{c}
| & 3.250 \\
+ & 0.041 \\
\hline
| 3.291 \\
\end{array}
\]

Practice problems

1) 1.004 + 32.1

2) 6.123 + 2 + 605.4

Answers: 1) 33.104; 2) 613.523

4.2.2 Subtract Decimals

Example

Subtract: 3.25 − 0.041

You may add zeros to the right so each number has the same length. Subtract:

\[
\begin{array}{c}
| 3.250 \\
- & 0.041 \\
\hline
| 3.209 \\
\end{array}
\]

\[
\begin{array}{c|c}
| 76.3 \\
- & 34.1 \\
\hline
| 42.2 \\
\end{array}
\]

\[
\begin{array}{c|c}
| 4.321 \\
- & 4.1 \\
\hline
| 0.221 \\
\end{array}
\]
4.3 Multiplying and Dividing Decimals

4.3.1 Multiply Decimals

Example

Multiply: \( 42.11 \times 0.2 \)

Ignoring the decimal point, first multiply

\[
\begin{array}{c}
4211 \\
\times 2 \\
\hline
8422
\end{array}
\]

Second, count the numbers to the right of the decimals. There are 2 in 42.11 and 1 in 0.2, so 3 total. Move the decimal point to the left 3 spots in the answer:

8.422

Practice problems

1) \( 605.23 \times 1.32 \)

2) \( 1.2 \times 7.13 \)

Answers: 1) 798.9036; 2) 8.556
4.3.2 Divide Decimals

**Example**

Divide: $6.515 \div 0.5$

$$
0.5 \overline{6.515}
$$

Move the decimal one hop to the right

$$
5 \overline{65.15}
$$

Perform the long division, lining up the decimal point in the answer

$$
\begin{array}{r}
13.03 \\
5 \quad 65.15 \\
5 \\
15.15 \\
15 \\
.15 \\
.15 \\
0
\end{array}
$$

So, $6.515 \div 0.5 = 13.03$

**Practice problems**

1) $0.539 \div 0.11$

2) $4.8 \div 0.6$

Answers: 1) 4.9; 2) 8
5 Ratios, Rates and Proportions

5.1 Ratio and Rates

5.1.1 Write a ratio in simplest form

Example

There are 15 yellow marbles and 35 red marbles. The ratio of yellow marbles to red marbles is 15 to 35. Ratios can be written using a colon 15:35 or in fraction form \(\frac{15}{35}\). Write the ratio in simplest form by reducing the fraction to lowest terms.

\[
\frac{15}{35} = \frac{3}{7}
\]
The ratio of yellow marbles to red marbles is 3:7.

Practice problems

1) There are 70 boys and 62 girls. Find the ratio of boys to girls in simplest form.

2) There are 27 bunnies and 81 hamsters. Find the ratio of bunnies to hamsters in simplest form.

Answers: 1) 35:31; 2) 1:3

5.1.2 Find a unit rate

Unit Rate

Unit Rate is Expressed as a Quantity of 1
A 12-ounce can of corn costs 69¢, the rate is 69¢ for 12 ounces.
Cost per ounce (unit) is \(\frac{12 \times 69}{12} = .17\)

Comparison to 1 Unit
Example

A service company charges $270 to clean 9 offices. What is the company's price for cleaning a single office?

\[
\frac{270}{9 \text{ offices}} = \frac{270 \div 9}{9 \div 9} = \frac{30}{1 \text{ office}}.
\]

The company charges $30 per office.

Practice problems

Find the unit rate:

1) 140 miles in 7 hours

2) $120 for 6 shirts

Answers: 1) 20 mph; 2) $20 per shirt

5.2 Proportions

5.2.1 Find an unknown in a proportion

<table>
<thead>
<tr>
<th>Problem</th>
<th>It takes Sandra 1 hour to word process 4 pages. At this rate, how long will she take to complete 27 pages?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Set up a proportion comparing the pages she types and the time it takes to type them.</td>
</tr>
<tr>
<td>4 pages</td>
<td>27 pages</td>
</tr>
<tr>
<td>1 hour</td>
<td>x hours</td>
</tr>
<tr>
<td>4 * x  = 1 * 27</td>
<td>Cross multiply.</td>
</tr>
<tr>
<td>4x = 27</td>
<td>You are looking for a number that when it is multiplied by 4 will give you 27.</td>
</tr>
<tr>
<td>( \frac{6.75}{4} )</td>
<td>You can find this value by dividing 27 by 4.</td>
</tr>
<tr>
<td>x = 6.75</td>
<td></td>
</tr>
<tr>
<td>Answer</td>
<td>It will take Sandra 6.75 hours to complete 27 pages.</td>
</tr>
</tbody>
</table>

Example

Three apples cost $3.99. How much does ten apples costs?

\[
\frac{3.99}{3 \text{ apples}} = \frac{x}{10 \text{ apples}}
\]

\[3.99 \times 10 = 3x\]

\[39.9 = 3x\]

$13.30. It costs $13.30 for 10 apples.
**Practice problems**

A recipe for oatmeal cookies calls for 3 cups of flour for every 4 cups of oatmeal.

1) How much oatmeal is needed if you plan to use only 2 cups of flour?

2) How much flour is needed if you plan to use 12 cups of oatmeal?

Answers: 1) $2 \frac{2}{3}$ cups of oatmeal; 2) 9 cups of flour

---

### 6 Percents

#### 6.1 Percent, Decimals, and Fractions

**6.1.1 Represent a number as a decimal, percent, and fraction**

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{3}{4}$</td>
<td>0.75</td>
<td>75%</td>
</tr>
</tbody>
</table>

**Example**

Convert 62% into a simplified fraction.

62% means 62 per 100. Written as a fraction: $\frac{62}{100} = \frac{31}{50}$

Convert 62.4% into a decimal.

Divide 62.4 by 100 (or move the decimal two spots to the left): 62.4% becomes 0.624

**Practice problems**

Convert 40% into:

1) a simplified fraction

2) a decimal

Answers: 1) $\frac{2}{5}$; 2) 0.40
6.1.2 Solve a percent problem

**Example**

25 is what percent of 400?

\[ 25 = 400 \times \]

\[ \frac{25}{400} = x \]

0.0625 = x

0.0625 \times 100 = 6.25%

**Practice problems**

1) What is 13% of 500?
2) 12 is what percent of 500?
3) 2 is 4% of what number?

Answers: 1) 65; 2) 2.4%; 3) 50

7 Real Numbers

7.1 Introduction to Real Numbers

7.1.1 Real Numbers

<table>
<thead>
<tr>
<th>Real Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rational</td>
</tr>
<tr>
<td>Integers</td>
</tr>
<tr>
<td>Whole</td>
</tr>
<tr>
<td>Natural</td>
</tr>
<tr>
<td>Irrational</td>
</tr>
</tbody>
</table>
Example

Identify which of the following numbers are integers:

$4, 0.5, -3, \frac{9}{7}, 0, 100$

Integers are $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$

0.5 is a decimal so it is not an integer, it is a rational number. $\frac{9}{7}$ is a fraction so it is not an integer, it is a rational number. 4, $-3$, 0, and 100 are integers.

Practice problems

1) Identify which of the following numbers are integers: $-6, 20, 1.1, -5.5, \frac{1}{2}, \frac{3}{2}, 0$

2) Identify which of the following numbers are irrational: $-1, 0, \sqrt{2}, \sqrt{4}$

Answers: 1) $-6, 20, 0$; 2) $\sqrt{2}$

7.1.2 Locate integers on a number line

The set of integers are represented on this number line.

Example

Plot the following integers on a number line:

A. $-9$
B. 4
C. 7
D. 0
E. $-2$
Practice problems

1) Plot $-5$ on a number line.

2) Plot $-1$ on a number line.

Answers:

7.1.3 Find the absolute value of a number

The symbol $|$ surrounding a number means **absolute value**. The absolute value of a number is always positive because it means the distance of the number from 0. For example $|-4|$ is 4 because $-4$ is 4 units away from 0.

The **opposite** of a number is the same distance from 0 on the number line, but on the opposite side of 0. For example, the opposite of $5$ is $-5$.

**Example**

Find $|-5|$.

The absolute value always return a positive version of a number.

So $|-5| = 5$

**Practice problems**

1) Find $|13|$.

2) Find the opposite of 13.

Answers: 1) 13; 2) $-13$

7.1.4 Compare integers

**Example**

Write the following integers from least to greatest: $-9$, 6, 0, $-1$, 3.

First you need to draw a number line and find each integer on the number line.
The integers to the left are less than integers to the right, so the integers in order from least to greatest is: $-9, -1, 0, 3, 6$

**Practice problems**

Insert the correct inequality symbol between the integers:

1) $-6 \underline{\quad} 6$  
2) $(-105) \underline{\quad} (-120)$

Answers: 1) $-6 < 6$; 2) $-105 > -120$

### 7.2 Operations with Real Numbers

#### 7.2.1 Add Integers

**Adding** two numbers with the **same sign**. For example $-5 + (-3) = -8$

1. Find the absolute values of each number
2. Add the absolute values
3. The sum takes the common sign of the two numbers

**Adding** two numbers with **different signs**. For example $-5 + 3 = -2$

1. Find the absolute values of each number
2. Subtract the smaller number from the larger number
3. The difference takes the sign of the larger of the two numbers

**Example**

Add $-4 + (-3)$

The solution is $-7$

If we add integers with like signs (both either negative or positive), we add the integers and keep the sign. The sum of two positive integers is a positive integer and the sum of two negative integers is a negative integer.

**Practice problems**

Add:

1) $-4 + 6$
2) $1 + (-9)$

Answers: 1) 2; 2) $-8$
### 7.2.2 Subtract Integers

**Subtracting** two numbers. For example \(-5 - (-3) = -2\)

- *If a and b are numbers, then* \(a - b = a + (-b)\).*
- *If a and b are numbers, then* \(a - (-b) = a + b\).*

#### Example

1. \(21 - (-5)\)  
   *is* \(21 + 5 = 26\)
2. \(-21 - (-5)\)  
   *is* \(-21 + 5 = -16\)
3. \(-21 - 5\)  
   *is* \(-21 + (-5) = -26\)
4. \(21 - 5\)  
   *is* \(21 + (-5) = 16\)

**Practice problems**

Subtract:

1) \(-101 - (-50)\)

2) \(101 - 50\)

*Answers:* 1) \(-51\); 2) \(51\)

### 7.2.3 Multiply and Divide Integers

- **Multiplying or dividing** two numbers with the *same sign* is a *positive* number
- **Multiplying or dividing** two numbers with *different signs* is a *negative* number. Note: Division by zero is undefined.

#### Example

\(-7 \times -8 = 56\) (*negative \times negative = positive*)

\(8 \div -2 = -4\) (*positive \div negative = negative*)

**Practice problems**

Multiply or divide:

1) \(-22 \times 3\)

2) \(-50 \div -2\)

*Answers:* 1) \(-66\); 2) \(25\)
7.2.4 **Order of Operations (PEMDAS)**
1. Starting with the innermost grouping symbol, perform all operations within the grouping symbols (such as parenthesis, brackets, and absolute value bars).
2. Evaluate all exponents.
3. Moving from left to right, perform all multiplying or dividing operations.
4. Moving from left to right, perform all adding or subtracting operations.

**Example**

\[ 4^2 - 6 \times 2 + 1 \]

1. Evaluate the exponent
   \[ 16 - 6 \times 2 + 1 \]
2. Multiply
   \[ 16 - 12 + 1 \]
3. Moving left to right, subtract and add.
   \[ 5 \]

**Practice problems**

1) \[ 6 + (2^2 + 5) \div 9 \]
2) \[ 8 + 3 \times (2 - 7)^2 \]

Answers: 1) 7; 2) 83

### 7.3 **Expressions and Properties**

<table>
<thead>
<tr>
<th>Properties of Real Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Property</strong></td>
</tr>
<tr>
<td>Commutative Property</td>
</tr>
<tr>
<td>Associative Property</td>
</tr>
<tr>
<td>Distributive Property</td>
</tr>
<tr>
<td>Identity Property</td>
</tr>
<tr>
<td>Inverse Property</td>
</tr>
</tbody>
</table>

#### 7.3.1 Evaluating Expressions

An **algebraic expression** is a mathematical expression that can contain variables (like \(x\) or \(y\)), numbers, and operations (like add, subtract, multiply, and divide).
A term in an algebraic expression can contain a variable and a numerical coefficient. For example, $3x + 2y - z - 1$ is an algebraic expression. The terms are $3x$, $2y$, $-z$, and $-1$. The variables are $x$, $y$, and $z$ and the numerical coefficients are $3$, $2$, and $-1$.

**Example**

Evaluate $x + 4$ when $x = 11$.

Substitute $11$ for $x$ and simplify

$11 + 4 = 15$

**Practice problems**

1) Evaluate $x + 1 - y - 7$ when $x = 3, y = -3$

2) Evaluate $x + 2y - 5$ when $x = 0, y = 5$

Answers: 1) 0; 2) 5

### 7.3.2 Simplifying Algebraic Expressions

**Like terms** are terms whose variables are the same. For example, $3x$ and $x$ are like terms. $3x$ and $3y$ are not like terms.

**Simplifying** an algebraic expression results in an equivalent expression that is simpler than the original. This is usually done by combining like terms or using the distributive property or other mathematical manipulations that make the expression simpler.

**Example**

Simplify:

$4(x + 2) + 3$

$4x + 8 + 3$ Distribute $4(x + 2)$ to remove the parenthesis

$4x + 11$ Combine like terms

**Practice problems**

Simplify:

1) $6(x - 3) + 4(x + y) - 3$

2) $5(2x + 11) - 4(x + 2)$

Answers: 1) $10x - 21 + 4y$; 2) $6x + 47$
8 Linear Equations

8.1 Solving Equations

8.1.1 Solving one-step equations

The **addition property of equality** states that

- if \( a = b \) then \( a + c = b + c \)
- if \( a = b \) then \( a - c = b - c \)

The **multiplication property of equality** states that

- if \( a = b \) then \( ac = bc \)
- if \( a = b \) then \( \frac{a}{c} = \frac{b}{c}; c \neq 0 \)

**Example**

Example 1: Solve \( x + 5 = 8 \)

We want to find the value of \( x \) such that it satisfies the equation. The goal is to get the variable \( x \) all by itself on one side of the equal sign. We keep \( x \) in one side and move 5 to the other side of the equation by subtracting 5 from each side.

\[
x + 5 - 5 = 8 - 5
\]

By simplifying the above equation we get

\[
x = 3.
\]

Example 2: Solve \( 3x = 15 \)

We want to find the value of \( x \) such that it satisfies the equation. The goal is to get the variable \( x \) all by itself on one side of the equal sign. We keep \( x \) in one side and move 3 to the other side of the equation by dividing each side by 3.

\[
\frac{3x}{3} = \frac{15}{3}
\]

By simplifying the above equation we get

\[
x = 5.
\]
Practice problems

Solve:

1) \(6 + y = -15\)
2) \(-15 = -7t\)

Answers: 1) \(y = -21\); 2) \(t = \frac{15}{7}\)

8.1.2 Solving multi-step equations

The goal to solving an equation is to get the variable alone on one side of the equation.

1. Simplify each side of the equation by using the distributive property to remove parenthesis and by combining like terms.
2. Use the addition property of equality to gather the variable terms on one side of the equation.
3. Use the addition property of equality to gather the constant terms on the other side of the equation.
4. Use the multiplication property of equality to divide both sides by the coefficient of the variable.
5. Check your solution.

Example

Solve:

\(3x - 7x + 10 + x = 6x - 4\)

We simplify each side by adding or subtracting like terms. Then we bring all terms with \(x\) in one side and all numbers to the other side.

\[-3x + 10 = 6x - 4\]  \(\text{Simplify each side of the equation by combining like terms.}\)

\[-3x + 3x + 10 = 6x + 3x - 4\]  \(\text{Gather the variable terms by adding } 3x \text{ to both sides}\)

\[10 = 9x - 4\]  \(\text{Simplify each side of the equation by combining like terms.}\)

\[10 + 4 = 9x - 4 + 4\]  \(\text{Add } 4 \text{ to both sides}\)

\[14 = 9x\]  \(\text{Simplify}\)

\[\frac{14}{9} = x\]  \(\text{Divide both sides by } 9\)
Practice problems

Solve:

1) \(18 + 5x - 7 = 4x - 6x + x - 1\)

2) \(3x - 4x + 20 - x = 30\)

Answers: 1) \(x = -2\); 2) \(x = -5\)

8.1.3 Solving Equations Containing Fractions

To solve an equation containing fractions, **clear the fractions** by multiplying both sides of the equation by the least common denominator of the fractions. For example, to solve \(\frac{3}{4}x + 2 = \frac{1}{6}\), begin by multiplying both sides by 12 to get \(9x + 24 = 2\).

**Example**

Solve for \(x\):

\[
x + \frac{x - 1}{7} = 3
\]

Multiply both side of the equation by least common denominator of 2 and 7 is 14.

\[
14 \left(\frac{x}{2} + \frac{x - 1}{7}\right) = 14(3)
\]

\[
7x + 2(x - 1) = 42
\]

\[
7x + 2x - 2 = 42
\]

\[
9x - 2 = 42
\]

\[
9x = 44
\]

\[
x = \frac{44}{9}
\]
Practice problems

Solve:

1) \( \frac{x}{6} + \frac{5x}{2} = \frac{1}{9} \)

2) \( \frac{x+2}{3} + \frac{x-1}{5} = 2 \)

Answers: 1) \( \frac{1}{24} \); 2) \( \frac{23}{8} \)

8.1.4 Solving special cases (no solution and infinite solution)

Example

Example 1: Solve \( 3x - 1 = 3x + 5 \)

We subtract \( 3x \) from both sides, we get \(-1 = 5\). Since \(-1 \neq 5\), we conclude that this equation does not have any solution.

Example 2: Solve \( 2x + 1 = 2x + 7 - 6 \)

We subtract \( 2x \) from both sides and combine like terms, we get \( 1 = 1 \). Since \( 1 = 1 \), this equation is true for any values of \( x \) and we conclude there is an infinite number of solutions.

Practice problems

Solve:

1) \( 4x - 5x + 11 = 6 + 5 - x \)

2) \( 2(x + 3) = 2x - 18 \)

Answers: 1) infinite solutions; 2) no solution

8.2 Formulas and Problem Solving

8.2.1 Evaluating a formula

Example

The formula used to convert temperature from Celsius to Fahrenheit is: \( F = \frac{9}{5} C + 32 \).

Express 35 Celsius as a temperature in degree Fahrenheit using the formula.
The value of C is given. We substitute it into the formula and simplify

\[ F = \frac{9}{5}(35) + 32 \]

\[ F = 95 \]

**Practice problems**

1) The area of a rectangle can be obtained by the formula \( A = lw \). Find \( A \) when \( l = 8 \text{ ft} \) and \( w = 9 \text{ ft} \).

2) The area of a triangle can be obtained by the formula \( A = \frac{1}{2}bh \). Find \( A \) when \( b = 5 \text{ cm} \) and \( h = 4 \text{ cm} \).

**Answers:** 1) \( A = 72 \text{ ft}^2 \); 2) \( A = 10 \text{ cm}^2 \)

### 8.2.2 Solving a formula for a variable

The goal in solving an equation containing more than one variable is to get the “solved for” variable alone on one side of the equation. To do this, use the same steps for solving a linear equation.

**Example**

Solve \( p = 2l + 2w \) for \( l \).

Since we want to solve the above equation for \( l \), we should keep it in one side and move \( 2w \) to the other side.

\[ p - 2w = 2l, \]

divide both sides by 2:

\[ \frac{p - 2w}{2} = l \]

**Practice problems**

1) Solve \( m = \frac{a+b}{2} \) for \( a \)

2) Solve \( 3x + 4y = 12 \) for \( y \)

**Answers:** 1) \( a = 2m - b \); 2) \( y = -\frac{3}{4}x + 3 \)
8.3 Inequalities

Solving Inequalities

To solve an inequality, use inverse operations and solve it just like an equation.

Extra Rule: If you multiply or divide both sides by a negative when doing inverse operations, you must reverse the inequality symbol.

The graph is an open circle and arrow if only less than or greater than, or a closed circle and arrow if also equal to.

8.3.1 Solving one-step inequalities

Example

Solve:
\[ x + 5 < -4 \]

\[ x + 5 - 5 < -4 - 5 \quad \text{Subtract 5 from both sides} \]

\[ x < -9 \]

Solve:
\[ -4x > 8 \]

\[ \frac{-4x}{-4} > \frac{8}{-4} \quad \text{Divide both sides by } -4 \]

\[ x < -2 \quad \text{Reverse the inequality symbol because we divided both sides by a negative.} \]

Practice problems

Solve:

1) \[ x - 7 > 6 \]

2) \[ 18 < 3x \]

Answers: 1) \( x > 13 \); 2) \( x > 6 \)
8.3.2 Solving multi-step inequalities

Example

Solve:

\[12x - 3x + 19 < 6 + 11x\]

\[9x + 19 < 6 + 11x\]
Combine like terms

\[-2x + 19 < 6\]
Subtract \(11x\) from both sides

\[-2x < -13\]
Subtract 19 from both sides

\[x > \frac{13}{2}\]
Divide both sides by \(-2\) and reverse the inequality symbol.

Practice problems

Solve:

1) \(3x + 6 - x < 12\)

2) \(4x - 8x + 5 > x - 1\)

Answers: 1) \(x < 3\); 2) \(x < \frac{6}{5}\)

8.3.3 Solving compound inequalities

<table>
<thead>
<tr>
<th>Compound Inequalities</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>WORDS</strong></td>
</tr>
<tr>
<td>All real numbers greater than 2 AND less than 6</td>
</tr>
<tr>
<td>All real numbers greater than or equal to 2 AND less than or equal to 6</td>
</tr>
<tr>
<td>All real numbers less than 2 OR greater than 6</td>
</tr>
<tr>
<td>All real numbers less than or equal to 2 OR greater than or equal to 6</td>
</tr>
</tbody>
</table>
Example

Solve:

\[-4 < 2x + 2 < 10\]

\[-4 - 2 < 2x + 2 - 2 < 10 - 2 \quad \text{Subtract 2 from all three sides}\]

\[-6 < 2x < 8\]

\[-\frac{6}{2} < \frac{2x}{2} < \frac{8}{2} \quad \text{Divide all three sides by 2}\]

\[-3 < x < 4\]

Practice problems

Solve:

1) \(5x < -10 \text{ or } x - 25 > 0\)

2) \(x \geq -10 + 12 \text{ and } 3x - 4 \leq 14\)

Answers: 1) \(x < -2 \text{ or } x > 25\); 2) \(2 \leq x \leq 6\)

9 Graphing Linear Equations

9.1 Graphs and Applications

9.1.1 Plot ordered pairs on a coordinate plane

Example

On a coordinate plane, label the x- and y-axes, label the Quadrants, and plot these points \((0,0), (3,5), (-3,5), (-3, -5), (3, -5)\)
**Practice problems**

Find the following points on a coordinate plane:

\((3,4), (5,2), (7, -4), \text{and } (-5, -6)\)

**Answers:**

**9.1.2 Graph a linear equation by finding ordered pairs**

All solutions to a linear equation with two variables can be represented by a line drawn on a coordinate system. Find at least two ordered pairs that satisfy the equation and connect the points.

**Example**

Graph the line \(y = x - 1\)

First, evaluate \(y\) for different values of \(x\). Complete the table:

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-2</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Second, plot the points on a coordinate plane and connect the dots with a line.

\(y = x - 1\)
**Practice problems**

1) Graph $y = 2x + 1$

2) Graph $y = -2x$

**Answers:**

$y = 2x + 1$

$y = -2x$

---

**9.1.3 Graph a linear equation by finding intercepts**

The intercepts of a graph are where the graph intersects the axes. The $x$-intercept is the point $(a, 0)$ and the $y$-intercept is the point $(0, b)$.

**Example**

Graph $y = x + 1$

Complete the table finding the points $(a, 0)$ and $(0, b)$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Plot the two points and connect the line.
Practice problems

1) Graph \( y = -x + 6 \)

2) Graph \( y = 4x - 5 \)

Answers: \( y = -x + 6 \) \( y = 4x - 5 \)

9.2 Slope

9.2.1 Finding the slope of a line from a graph

The slope \( (m) \) of a line that goes through point \((x_1, y_1)\) and point \((x_2, y_2)\) is:

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

Example

Find the slope of the following line:

Identify two points on the line. Use the slope formula to find the slope.

\((x_1, y_1) = (3, 4)\) and \((x_2, y_2) = (5, 2)\)

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 4}{5 - 3} = \frac{-2}{2} = -1
\]
Practice problems

1) Find the slope of a line that passes through the points \((-1, 9)\) and \((3, 8)\)

2) Find the slope of a line that passes through the point \((5, -5)\) and \((10, 1)\)

Answers: 1) \(-\frac{1}{4}\); 2) \(\frac{6}{5}\)

9.2.2 Find the slope of a line from an equation

Slope-intercept form of a line: \(y = mx + b\) where \(m\) is the slope and \((0, b)\) is the y-intercept.

Standard form of a line: \(Ax + By = C\) where \(A\), \(B\), and \(C\) are real numbers and \(A\) and \(B\) are not both equal to zero.

Example

Find the slope of the line \(y + x = 1\)

To find the slope of a line, we first have to bring the equation into this form \(y = mx + b\), then the slope is \(m\).

In order to bring \(y + x = 1\) into the above mentioned form we subtract \(x\) from both sides:

\[y = -x + 1,\]

The slope is the coefficient of \(x\) which is \(-1\).

Practice problems

1) Find the slope of \(2y = 4x - 8\)

2) Find the slope of \(y - 3x = 7\)

Answers: 1) \(m = 2\); 2) \(m = 3\)

9.2.3 Find slopes of horizontal and vertical lines

- Horizontal lines have a slope of 0
- Vertical lines have an undefined slope
- The equation of a horizontal line is \(y = b\) where \((0, b)\) is the y-intercept.
- The equation of a vertical line is \(x = a\) where \((a, 0)\) is the x-intercept.

Example

Find the slopes of the lines \(y = 1\) and \(x = -2\)
For $y = 1$, $x$ can be any value, but $y$ is the fixed value 1. So, it is a horizontal line and the slope of this line is zero because the changes in $y$ is zero (it never moves up or down).

$$slope = \frac{\text{changes in } y}{\text{changes in } x} = \frac{0}{\text{changes in } x} = 0.$$ 

For $x = -2$ the $x$ value is fixed at $-2$ and the $y$ value can be anything, so it is a vertical line. The slope of a vertical line is undefined because the changes in $x$ is zero (never moves to the left or right)

$$slope = \frac{\text{changes in } y}{\text{changes in } x} = \frac{\text{changes in } y}{0} = \text{undefined}.$$ 

**Practice problems**

1) Find the slope of $y = -13$

2) Find the slope of $x = 0$

Answers: 1) 0; 2) undefined

---

**10 Linear Modeling**

**10.1 Equations of lines**

**10.1.1 Find the equation of a line given slope and $y$ intercept**

**Example**

Find the equation of a line in slope-intercept form that has a slope of $-1$ and a $y$-intercept of $(0,4)$.

The slope-intercept form of the equation of a line is $y = mx + b$ where $m$ is the slope and $b$ is the $y$ value of the $y$-intercept.

Therefore, the equation of a line with a slope of $-1$ and a $y$-intercept of $(0,4)$ is $y = -x + 4$.

**Practice problems**

1) Find the equation of a line that passes through the points $(1,6)$ and $(-5,3)$ and has the $y$-intercept of $(0,-2)$. [Hint: first use the slope formula to find the slope of the line]

2) Find the equation of a line that has a slope of $-\frac{2}{3}$ and a $y$-intercept of $(0,5)$.

Answers: 1) $y = \frac{1}{2}x - 2$; 2) $y = -\frac{2}{3}x + 5$
10.1.2 Find the equation of line given slope and a point

**Point-slope form** of a line: \( y - y_1 = m(x - x_1) \) where \( m \) is the slope and \((x_1, y_1)\) is a point on the line.

**Example**

Find the equation of a line in slope-intercept form that passes through the point \((1,2)\) and has a slope of 7.

Use the point-slope form \( y - y_1 = m(x - x_1) \) and substitute \( m = 7, x_1 = 1, y_1 = 2 \).

\[
\begin{align*}
y - 2 &= 7(x - 1) \\
y - 2 &= 7x - 7 & \text{Distribute the 7} \\
y &= 7x - 5 & \text{Add 2 to both sides}
\end{align*}
\]

**Practice problems**

1) Find the equation of a line that passes the point \((-1,5)\) and has a slope of -2.

2) Find the equation of a line that passes the point \((3,-5)\) and has a slope 0.

Answers: 1) \( y = -2x + 3 \); 2) \( y = -5 \)

10.1.3 Find the equation of a line given two points

**Example**

Find the equation of a line in slope-intercept form passing through \((-2,3)\) and \((-1,4)\).

First, find the slope \( m = \frac{y_2-y_1}{x_2-x_1} = \frac{3-(-5)}{-1-(-2)} = \frac{8}{1} = 8 \).

\[
m = \frac{4 - 3}{-1 + 2} = \frac{1}{1} = 1
\]

Then, use the point-slope formula to substitute the slope, and one of the points. We will pick the point \((-2,3)\)

\[
y - 3 = 1(x + 2)
\]

\[
y = x + 5
\]
10.1.4 Find the slope of a parallel line

Parallel lines have the same slope.

Example

Find the equation of the line that is parallel to \( y = 2x - 1 \) and passes through the point \((2, -3)\).

The parallel lines have the same slope. The given line is in slope-intercept form \( y = mx + b \), so we can discern that the slope of the given line is 2. Therefore, the slope of the line that we want to find is also 2. Begin by writing the equation of the line in point-slope form is and then solving for \( y \):

\[
y + 3 = 2(x - 2)
\]
\[
y + 3 = 2x - 4
\]
\[
y = 2x - 7
\]

Practice problems

1) Find the equation of the line that is parallel to \( y = -4x + 7 \) and has \( y \)-intercept 2.

2) Is \( y = x - 4 \) parallel to \( 2y - 2x = -8 \)?

Answers: 1) \( y = -4x + 2 \); 2) yes

10.1.5 Find the slope of a perpendicular line

If one line has the negative reciprocal slope of another line, then the two lines are perpendicular.

Example

Find the equation of the line in slope-intercept form that is perpendicular to \( y = 4x - 3 \) and has the \( y \)-intercept \((0, -5)\).
When one line has a slope of $m$, a perpendicular line has a slope of $-\frac{1}{m}$. The given line has $m = 4$, so the slope of the perpendicular line is $-\frac{1}{4}$. The equation of the line is:

$$y = -\frac{1}{4}x - 5.$$ 

**Practice problems**

1) Find the equation of the line that is perpendicular to $y = -4x + 1$ and passes through (1,2).

2) Are these two lines perpendicular: $y = 2x + 7$, and $y = -0.5x - 1$?

Answers: 1) $y = \frac{1}{4}x + \frac{7}{4}$; 2) yes

**11 Systems of Equations**

**11.1 Solving Systems of Equations**

When two lines are graphed on the same coordinate graph, there are three possibilities:

- The two lines will intersect at one point. This is called a **consistent** system and the equations are **independent**.
- The two lines are the same line. These lines intersect at every point on the lines, so there are an infinite number of solutions. This is called a **consistent** system and the equations are **dependent**.
- The two lines will never intersect, so there is no solution. In other words, the lines are parallel. This is called an **inconsistent** system.

Solving a system of two linear equations means:

- Finding the ordered pair where the two lines intersect, or
- Discovering that the two lines are either parallel or the same line.

**11.1.1 Solving Systems of Linear Equations by Graphing**

1. Graph both lines on the same coordinate plane.
2. If the lines intersect, the solution is the ordered pair of intersection.
3. If the lines are parallel, there is no solution.
4. If the lines are the same line, the solution is all the points on the lines so there are an infinite number of solutions.
Example

Solve the system by graphing:

\[
\begin{align*}
-3x + y &= -4 \\
y + 2x &= 1
\end{align*}
\]

Step 1: Graph the two lines.

Step 2: Find the point of intersection. The solution is \((1, -1)\).

Practice problems

1) Find the solution of \(y = x - 1\) and \(y = 2x + 1\).

2) Find the solution of \(y = 2x - 4\) and \(y = 2x + 6\).

Answers: 1) \((-2, -3)\); 2) no solution

11.1.2 Solving Systems of Linear Equations by Substitution

1. If needed, solve one equation for one of the variables. For instance, solve one equation for \(y\) in terms of \(x\).
2. Substitute the expression for \(y\) found in step 1 into the other equation.
3. Solve the equation from step 2 for \(x\).
4. Using the solution from step 3, plug it into either of the original equations to find the answer for the second variable, \(y\).
5. State the solution as an ordered pair \((x, y)\).
Note: It may be easier to solve one equation for \( x \) in terms of \( y \) in step 1. In that case, step 2 will be to substitute the expression for \( x \) found in step 1 into the other equation. Step 3 gives you an answer for \( y \). Step 4 gives you an answer for \( x \).

**Example**

Solve the system by substitution:

\[
\begin{align*}
\begin{cases}
  x + y &= 1 \\
  4x + 7y &= 10
\end{cases}
\end{align*}
\]

Steps:

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>If needed, solve one equation for one of the variables. In other words, solve one equation for ( y ) in terms of ( x ).</td>
<td>( x + y = 1 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( x - x + y = 1 - x )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( y = 1 - x )</td>
</tr>
<tr>
<td>2.</td>
<td>Substitute the expression for ( y ) found in step 1 into the other equation.</td>
<td>( 4x + 7y = 10 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( 4x + 7(1 - x) = 10 )</td>
</tr>
<tr>
<td>3.</td>
<td>Solve the equation from step 2 for ( x ).</td>
<td>( 4x + 7(1 - x) = 10 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( 4x + 7 - 7x = 10 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( -3x + 7 = 10 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( -3x + 7 - 7 = 10 - 7 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( -3x = 3 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( -3x = \frac{3}{-3} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( x = -1 )</td>
</tr>
<tr>
<td>4.</td>
<td>Using the solution from step 3, plug it into either of the original equations to find the answer for the second variable, ( y ).</td>
<td>( x + y = 1 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( (-1) + y = 1 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( (-1) + 1 + y = 1 + 1 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( y = 2 )</td>
</tr>
<tr>
<td>5.</td>
<td>State the solution as an ordered pair ((x, y)).</td>
<td>The solution to the system, and where the two lines intersect, is the point ((-1, 2))</td>
</tr>
</tbody>
</table>
**Practice problems**

1) Find the solution of \( y = -x - 1 \) and \( y + 2x = 3 \)

2) Find the solution of \( 2y - x = 1 \) and \( y = 5x + 1 \)

**Answers:**

1) \((4, -5)\); 2) \((-\frac{1}{9}, \frac{4}{9})\)

11.1.3 Solving Systems of Linear Equations by Addition (or Elimination)

1. Both equations need to be written in standard form. \( Ax + By = C \)

2. Choose which variable to eliminate.

3. The coefficients of your chosen variable must be opposites (the same number but different signs). If necessary, multiply one or both equations by a number (that is not zero) to create this situation.

4. Add the equations. Your chosen variable will eliminate (add to zero).

5. Solve for the other variable.

6. Using the solution from step 5, plug it into either of the original equations to find the answer for the second variable.

7. State the solution as an ordered pair \((x, y)\).

**Example**

Solve the system by addition:

\[
\begin{align*}
4x + 2y &= 10 \\
x - y &= 1
\end{align*}
\]

| 1. Both equations need to be written in standard form. \( Ax + By = C \) | 4x + 2y = 10 \\
<table>
<thead>
<tr>
<th></th>
<th>x - y = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Choose which variable to eliminate.</td>
<td>Choose to eliminate ( y ).</td>
</tr>
</tbody>
</table>
| 3. The coefficients of your chosen variable must be opposites (the same number but different signs). If necessary, multiply one or both equations by a number (that is not zero) to create this situation. | Multiply the second equation by 2: \\
| | \( 2(x - y) = 2(1) \) \\
| | \( 2x - 2y = 2 \) |
| 4. Add the equations. Your chosen variable will eliminate (add to zero). | 4x + 2y = 10 \\
| | 2x - 2y = 2 \\
| | 6x + 0 = 12 |
5. Solve for the other variable.

\[
\begin{align*}
6x &= 12 \\
6x &= 12 \\
\frac{6x}{6} &= \frac{12}{6} \\
x &= 2
\end{align*}
\]

6. Using the solution from step 5, plug it into either of the original equations to find the answer for the second variable.

\[
\begin{align*}
x - y &= 1 \\
2 - y &= 1 \\
2 - 2 - y &= 1 - 2 \\
- y &= -1 \\
y &= 1
\end{align*}
\]

7. State the solution as an ordered pair \((x, y)\).

The solution to the system, and where the two lines intersect, is the point \((2,1)\).

### Practice problems

1) Solve the system by addition:

\[
\begin{cases}
2x + y = 10 \\
x - y = 5
\end{cases}
\]

2) Solve the system by addition:

\[
\begin{cases}
4x + 7y = 8 \\
2x - y = 6
\end{cases}
\]

Answers: 1) \((5,0)\); 2) \((\frac{25}{9}, \frac{-4}{9})\)

### 11.2 Solving Systems – no solutions

**Note on Substitution and Addition Methods**: If solving a system using either the substitution or addition methods gives a true statement, like \(3 = 3\), then the two lines are the same line and there are an infinite number of solutions. If the method gives a false statement, like \(0 = 3\), then the two lines are parallel and there is no solution.

### 11.2.1 Solve a no solution system algebraically

**Example**

\[
\begin{cases}
6x + 2y = 7 \\
y = 2 - 3x
\end{cases}
\]
1. If needed, solve one equation for one of the variables.  
   \[ y = 2 - 3x \]

2. Substitute the expression for \( x \) found in step 1 into the other equation.  
   \[ 6x + 2y = 7 \]  
   \[ 6x + 2(2 - 3x) = 7 \]

3. Solve the equation from step 2 for \( y \).  
   \[ 6x + 2(2 - 3x) = 7 \]  
   \[ 6x + 4 - 6x = 7 \]  
   \[ 4 = 7 \]  
   This is a false statement. Therefore, the lines are parallel and there is no solution.

If solving a system using either the substitution or addition methods gives a true statement, like \( 3 = 3 \), then the two lines are the same line and there are an infinite number of solutions. If the method gives a false statement, like \( 0 = 3 \), then the two lines are parallel and there is no solution.

**Practice problems**

1) Solve the system:  
   \[
   \begin{align*}
   y &= 2x - 1 \\
   y &= 2x + 10
   \end{align*}
   \]

2) Solve the system:  
   \[
   \begin{align*}
   y - x &= 4 \\
   y &= x
   \end{align*}
   \]

Answers: 1) no solution; 2) no solution
### 11.2.2 Solve an infinite solution system algebraically

**Example**

\[
\begin{align*}
-16x + y &= 2 \\
48x - 3y &= -6
\end{align*}
\]

Steps:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
</table>
| 1. Both equations need to be written in standard form. \(Ax + By = C\) | \(-16x + y = 2\)  
|   | \(48x - 3y = -6\) |
| 2. Choose which variable to eliminate. | Choose to eliminate \(y\). |
| 3. The coefficients of your chosen variable must be opposites (the same number but different signs). If necessary, multiply one or both equations by a number (that is not zero) to create this situation. | Multiply the first equation by 3:  
\[
3(-16x + y) = 3(2) \\
-48x + 3y = 6
\]  
|   |   |
| 4. Add the equations. Your chosen variable will eliminate (add to zero). | \(-48x + 3y = 6\)  
|   | \(48x - 3y = -6\)  
|   | \(0 + 0 = 0\)  
|   | \(0 = 0\)  
|   |   |
| If solving a system using either the substitution or addition methods gives a true statement, like \(3 = 3\), then the two lines are the same line and there are an infinite number of solutions. If the method gives a false statement, like \(0 = 3\), then the two lines are parallel and there is no solution. |   |
|   | This is a true statement. Therefore, the lines are the same line and there are **infinite solutions**. |

**Practice problems**

1) Solve the system:

\[
\begin{align*}
y &= x + 3 \\
2y - 2x &= 6
\end{align*}
\]
2) Solve the system:

\[
\begin{align*}
5x + y &= -1 \\
4y &= -20x - 4
\end{align*}
\]

Answers: 1) infinite solutions; 2) infinite solutions

11.3 Problem Solving

11.3.1 Systems of Linear Equations and Problem Solving

1. Read and reread the word problem to fully understand it. Draw pictures or diagrams.
2. Identify the unknowns and let each unknown be a separate variable. If we have two unknowns then we have two variables, \(x\) and \(y\).
3. Translate the words into two equations.
4. Solve the two equations using any method.
5. Interpret the results and check the solution.

Example

Find two numbers whose sum is 11 and whose difference is 5.

Solution:

Let \(x\) represent the first number and \(y\) represent the second number.
“…two numbers whose sum is 11” is translated \(x + y = 11\)
“…two numbers whose difference is 5” is translated \(x - y = 5\)

Solve the system of equations using any method. Since both equations are in standard form, the addition method will be used here.

\[
\begin{align*}
x + y &= 11 \\
x - y &= 5
\end{align*}
\]

\[
\begin{align*}
2x &= 16 \\
x &= 8
\end{align*}
\]

Substituting \(x = 8\) into the first equation and solving for \(y\):

\[
\begin{align*}
8 + y &= 11 \\
8 - 8 + y &= 11 - 8 \\
y &= 3
\end{align*}
\]

The two numbers are 8 and 3 because the sum of the two numbers is 11 and the difference of the two numbers is 5.
Practice problems

1) Find the value of two numbers if their sum is 70 and their difference is 15.

2) Stefan is selling tickets to a choral performance. On the first day of ticket sales Stefan sold 2 senior citizen tickets and 1 child ticket for a total of $50. On the second day, Stefan collected $110 by selling 5 senior citizen tickets and 2 child tickets. Find the price of a senior citizen ticket and the price of a child ticket.

Answers: 1) the two numbers are $\frac{85}{2}$ and $\frac{55}{2}$; 2) child ticket is $30 and a senior ticket is $10

12 Exponents

12.1 Introduction to exponents

Using exponents, multiplying the same value over and over again can be written in a more efficient form. For example, $3 \cdot 3 \cdot 3 = 3^4$ or $x \cdot x \cdot x = x^3$

12.1.1 Evaluate an expression with exponents

Example

Evaluate:

a) $(-5)^2$

b) $-4^2$

c) $(4x)^0$

d) $4x^2$ if $x = -3$

Answer:

a) $(-5)(-5) = 25$

b) $-(4)(4) = -16$

c) $(4x)^0 = 1$

Answers: 1) $138$; 2) $-17$
12.1.2 Evaluate an expression with negative exponents

\[ a^{-b} = \frac{1}{a^b} \]

**Example**

Evaluate:

a) \(4x^{-3}\) when \(x = 1\)

b) \(\left(\frac{x^2}{y}\right)^{-2}\) when \(x = 2\) and \(y = -2\)

Answer:

a) \(\frac{4}{x^3} = 4\)

b) \(\frac{x^{-4}}{y^{-2}} = \frac{y^2}{x^4} = \frac{(-2)^2}{(2)^4} = \frac{4}{16} = \frac{1}{4}\)

**Practice problems**

1) Evaluate \(3x^{-2} + 5y^{-3}\) when \(x = 1\) and \(y = 2\)

2) Evaluate \(2x^{-4} + y - 6\) when \(x = -2\) and \(y = 5\)

Answers: 1) \(\frac{29}{8}\), 2) \(-\frac{7}{8}\)

12.1.3 Evaluate an expression with rational exponents

**Example**

Evaluate \(8^{\frac{1}{3}}\)

Convert the expression with rational exponent to the radical equivalent. The denominator of the fraction determines the root, in this case the cube root.

\[ 8^{\frac{1}{3}} = \sqrt[3]{8} = 2.\]
### Practice problems

1) Evaluate $16^{\frac{1}{2}}$

2) Evaluate $81^{\frac{1}{2}}$

Answers: 1) 4; 2) 3

### 12.2 Rules of Exponents

Use the **Rules of Exponents** to simplify an algebraic term containing exponents. A **simplified term** has no negative exponents, no parentheses, and each variable appearing only once.

<table>
<thead>
<tr>
<th>The Rules of Exponents</th>
<th>Examples</th>
</tr>
</thead>
</table>
| **Zero**               | $a^0 = 1$  
  $a \neq 0$              | $(-20)^0 = 1$  
  $x^0 = 1$               |
| **Negative**           | $a^{-b} = \frac{1}{a^b}$  
  $\frac{1}{a^{-b}} = a^b$  
  $\frac{a^{-b}}{c^{-d}} = \frac{c^d}{a^b}$  
  $a \neq 0, c \neq 0$   | $x^{-4} = \frac{1}{x^4}$  
  $\frac{1}{3^{-2}} = 3^2 = 9$  
  $x^{-4} \cdot y^5 = \frac{x^4}{y^5}$ |
| **Product Rule**       | $a^b \cdot a^c = a^{b+c}$  
  $a$ is any real number | $5^2 \cdot 5^8 = 5^{10}$  
  $x^{-2} \cdot x^4 = x^2$ |
| **Quotient Rule**      | $\frac{a^b}{a^c} = a^{b-c}$  
  $a \neq 0$              | $\frac{2^3}{2^2} = 2^1 = 2$  
  $\frac{x^5}{x^2} = x^3$ |
| **Power Rule**         | $(a^b)^c = a^{b\cdot c}$  
  $(ab)^c = a^c b^c$  
  $\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$  
  $b \neq 0$             | $(z^4)^2 = z^8$  
  $(3x)^2 = 3^2 x^2 = 9x^2$  
  $\left(\frac{2x}{y}\right)^3 = \frac{8x^3}{y^3}$ |
12.2.1 Simplify using the Product rule

Example

Simplify:

\((-3x^3y^4)(5x^5y^3)\)

Multiply the coefficients and add the exponents

\(-15x^8y^7\)

Practice problems

1) Simplify \((14^6)(14^{-7})\)

2) Simplify \((3x^2)(5x^{19})\)

Answers: 1) \(\frac{1}{14}\); 2) \(15x^{21}\)

12.2.2 Simplify using the Quotient Rule

Example

Simplify:

\(-\frac{12x^9}{3x^5}\)

Divide the coefficients and subtract the exponents

\(-4x^4\)

Practice problems

1) Simplify \(\frac{15x^5}{5x^2}\)

2) Evaluate \(\frac{7^3}{7^2}\)

Answers: 1) \(3x^3\); 2) 7
12.2.3 Simplify using the Power Rule

**Example**

Simplify:

\[(4x^4y^3)^3\]

Multiply the exponents. Remember that the exponent on the coefficient 4 is 1.

\[64x^{12}y^9\]

**Practice problems**

1) Simplify \((3x^4y)^2\)

2) Simplify \(\left(\frac{x^4}{y^2}\right)^3\)

Answers: 1) \(9x^8y^2\); 2) \(\frac{x^{12}}{y^6}\)

12.3 Scientific Notation

Exponents are used when writing very large or very small positive numbers in **scientific notation**. For example, it would take a large calculator screen to read 263,400,000,000 or the very small number 0.0000000000000000000000679.

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>Scientific Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>263,400,000,000</td>
<td>(2.634 \times 10^{11})</td>
</tr>
<tr>
<td>0.000000000000000000679</td>
<td>(6.79 \times 10^{-23})</td>
</tr>
</tbody>
</table>

To express a positive number less than 1 in scientific notation, move the decimal point to the right until a number between 1 and 10 is created. Count the amount of steps the decimal point traveled to create this number. The amount of steps is your exponent on 10, expressed as a negative number.

To express a positive number greater than 1 in scientific notation, move the decimal point to the left until a number between 1 and 10 is created. Count the amount of steps the decimal point traveled to create this number. The amount of steps is your exponent on 10, expressed as a positive number.
12.3.1 Convert a number to scientific notation

*Example*

Write in scientific notation: 0.0000056784

Move the decimal point to the right 6 spaces

\[ = 5.6784 \times 10^{-6} \]

*Practice problems*

1) Convert to scientific notation 137.6
2) Convert to scientific notation 0.00049

Answers: 1) 1.376 \times 10^2; 2) 4.9 \times 10^{-4}

12.3.2 Convert from scientific notation to decimal

*Example*

Write in standard form: 4.12 \times 10^6

Move the decimal point to the right 6 spaces

\[ = 4,120,000 \]

*Practice problems*

1) Convert to decimal 3.5 \times 10^3
2) Convert 5.5 \times 10^{-4} to decimal

Answers: 1) 3500; 2) 0.00055

13 Polynomials

13.1 Introduction to Polynomials

A *polynomial* is an expression of summed algebraic terms with positive integer exponents. For example, \[ 3x^2 + 9y^2 - 1 \] is a polynomial. Some specific types of polynomials are:

- **monomials** = one term, like \(6x\)
- **binomials** = two terms, like \(4x - 9\)
- **trinomials** = three terms, like \(x^2 + 7x - 12\)
A polynomial is written in standard form when the terms are in descending exponential order. For example, $x^2 + 7x - 12$ is a trinomial in standard form.

The degree of a polynomial is the largest exponent appearing in the polynomial. The degree of $x^2 + 7x - 12$ is 2.

Simplifying a polynomial results in an equivalent expression that is simpler than the original. This is usually done by combining like terms or using the distributive property or other mathematical manipulations that make the expression simpler.

Like terms are terms whose variables and exponents are the same. For example, $3x^2$ and $x^2$ are like terms. $3x^2$ and $3x$ are not like terms.

### 13.1.1 Identify terms, coefficients, exponents and degree of a polynomial

**Example**

Write in standard form and find the degree of the polynomial

$-24x^3 + 9x^5 - 8x + 2$

Write the terms in descending exponential order

$9x^5 - 24x^3 - 8x + 2$

The degree of the polynomial is 5.

**Practice problems**

1) Find the leading coefficient and the degree of the polynomial $5x^3 - x + 1$

2) Find the leading coefficient and the degree of the polynomial $x^4 + 2x^3 - x + 6$

Answers: 1) leading coefficient 5, degree 3; 2) leading coefficient 1, degree 4

### 13.1.2 Evaluate a polynomial

**Example**

Evaluate $3x^2 - 7x + 20$ for $x = 2$

Substitute 2 for each $x$ in the polynomial and use the order of operations to simplify.

$3x^2 - 7x + 20$

$3(2)^2 - 7(2) + 20$

$18$
Practice problems

1) Evaluate $5y^3 - 6y + 7$ for $y = -1$
2) Evaluate $x^4 - 2x^2 + x - 8$ for $x = 3$

Answers: 1) 8; 2) 58

13.2 Operations with Polynomials

13.2.1 Add Polynomials
To add polynomials, combine like terms.

Example

Add the polynomial
$(-3x^3 + 4x^2 - 2) + (5x^3 + 3x + 2)$

Remove the parentheses and combine like terms:

$2x^3 + 4x^2 + 3x$

Practice problems

1) Add: $(11x^2 - x) + (15x^3 - 11x^2 + 10x - 1)$
2) Add: $(4x^4 - 5x + 2) + (-x^4 + 10x^2 - x + 6)$

Answers: 1) $15x^3 + 9x - 1$; 2) $3x^4 + 10x^2 - 6x + 8$

13.2.2 Subtract Polynomials
To subtract polynomials, distribute the negative sign on the second polynomial, then add.

Example

Subtract the polynomials
$(3x^3 + x^2 - 2) - (2x^3 - x^2 + 2)$

Distribute the negative sign, remove the parentheses and combine like terms:

$3x^3 + x^2 - 2 - 2x^3 + x^2 - 2$

$x^3 + 2x^2 - 4$
**Practice problems**

1) Subtract \((7x^3 - 5x + 2) - (6x^3 - 7x^2 + 1)\)

2) Subtract \((4x^4 - 1) - (x^4 + 4x^3 - x^2 + 5x)\)

Answers: 1) \(x^3 + 7x^2 - 5x + 1\); 2) \(3x^4 - 4x^3 + x^2 - 5x - 1\)

### 13.2.3 Multiply Polynomials

Multiply each term of the first polynomial to each term of the second polynomial. Then combine like terms.

When multiplying two binomials, the **FOIL** method is helpful.

- **F**: multiply the first terms of each binomial
- **O**: multiply the outside terms of each binomial
- **I**: multiply the inside terms of each binomial
- **L**: multiply the last terms of each binomial

Then combine like terms.

Some shortcuts:

**Sum and difference of two terms**: \((x - y)(x + y) = x^2 - y^2\)

**Squaring a binomial**: \((x + y)^2 = (x + y)(x + y) = x^2 + 2xy + y^2\)

\((x - y)^2 = (x - y)(x - y) = x^2 - 2xy + y^2\)

**Example**

Example 1: Multiply \((-3x)(-4x^2 + x - 2)\)

Use the distributive property to multiply each term of the polynomial by \(-3x\):

\(12x^3 - 3x^2 + 6x\)

Example 2: Multiply \((x + 4)(x^2 - 2x + 1)\)

Use the distributive property to multiply each term of the binomial by each term of the trinomial. Then combine like terms.

\(x(x^2 - 2x + 1) + 4(x^2 - 2x + 1)\)

\(x^3 - 2x^2 + x + 4x^2 - 8x + 4\)

\(x^3 + 2x^2 - 7x + 4\)
**Practice problems**

1) Multiply \((x + 4)(x + 1)\)
2) Multiply \((2x^2 - 4)(3x^2 + x - 4)\)

Answers: 1) \(x^2 + 5x + 4\); 2) \(6x^4 + 2x^3 - 20x^2 - 4x + 16\)

### 13.2.4 Divide Polynomials by a monomial

To divide a polynomial by a monomial:

\[
\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}, \quad c \neq 0
\]

**Example**

Divide \(\frac{20x^5 - 15x^3 + 5x^2 + 25x}{5x}\)

\[
\frac{20x^5}{5x} - \frac{15x^3}{5x} + \frac{5x^2}{5x} + \frac{25x}{5x}
\]

divide each term of the polynomial by the monomial \(5x\)

\[4x^4 - 3x^2 + x + 5\]

simplify each fraction

**Practice problems**

1) Divide \(\frac{25x^4 - 5x^3 + x^2 - 4x}{5x^2}\)
2) Divide \(\frac{15x^2y - xy^2 + 5}{5xy}\)

Answers: 1) \(5x^2 - x + \frac{1}{5} - \frac{4}{5x}\); 2) \(3x - \frac{y}{5} + \frac{1}{xy}\)
13.2.5 Divide Polynomials by a binomial (long division)

Use long division to divide a polynomial by something other than a monomial.

**Example**

Divide using long division:

\[
\frac{x^3 - 6x^2 + 7x - 2}{x - 1}
\]

\[
x - 1 \overline{x^3 - 6x^2 + 7x - 2}
\]

Divide the first terms: \(\frac{x^3}{x} = x^2\)

\[
x - 1 \overline{x^2 - 6x^2 + 7x - 2}
\]

Multiply \(x^2(x - 1)\):

\[
x^3 - x^2
\]

Subtract:

\[
-5x^2 + 7x - 2
\]

Repeat: Divide the first terms: \(\frac{-5x^2}{x} = -5x\)

\[
x - 1 \overline{x^2 - 5x + 2}
\]

Multiply \(-5x(x - 1)\):

\[
x^3 - x^2
\]

Subtract:

\[
-5x^2 + 5x
\]

Repeat: Divide the first terms: \(\frac{2x}{x} = 2\)

\[
x - 1 \overline{x^2 - 5x + 2}
\]

Multiply \(2(x - 1)\):

\[
x^3 - x^2
\]

Subtract:

\[
2x - 2
\]

Therefore,

\[
\frac{x^3 - 6x^2 + 7x - 2}{x - 1} = x^2 - 5x + 2
\]
**14 Factoring Polynomials**

### 14.1 Introduction to factoring

Factors divide evenly into a number or expression. For example, 6 divides evenly into 12, so 6 is a factor of 12. The binomial \((x + 2)\) divides evenly into the trinomial \(x^2 + 3x + 2\), so \((x + 2)\) is a factor of \(x^2 + 3x + 2\).

Factoring a polynomial is the process of finding and rewriting a polynomial expression as the product of its prime factors.

A prime factor, like a prime number, has no additional factors besides itself and 1.

#### 14.1.1 Factor out the GCF

1. Find the greatest common factor of all the terms in the polynomial.
2. Place the GCF outside a set of parentheses: \(\text{GCF}()\).
3. Divide each of the terms by the GCF and rewrite the new polynomial inside the parentheses.
4. Double check by distributing the GCF to verify that the product is the original polynomial.

**Example**

Factor the GCF from: \(10y^7 + 2y^3 + 6y^5\)

Answer: \(2y^3(5y^4 + 1 + 3y^2)\)

Explanation:

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Find the greatest factor in common of all the terms in the polynomial.</td>
<td>The GCF of (10y^7, 2y^3, ) and (6y^5) is (2y^3).</td>
</tr>
<tr>
<td>2</td>
<td>Place the GCF outside a set of parentheses: (\text{GCF}()).</td>
<td>(2y^3())</td>
</tr>
<tr>
<td>3</td>
<td>Divide each of the terms by the GCF and rewrite the new polynomial inside the parentheses.</td>
<td>(2y^3(5y^4 + 1 + 3y^2))</td>
</tr>
<tr>
<td>4</td>
<td>Double check by distributing the GCF to verify that the product is the original polynomial.</td>
<td>(2y^3(5y^4 + 1 + 3y^2) = 10y^7 + 2y^3 + 6y^5)</td>
</tr>
</tbody>
</table>
Practice problems

1) Factor $7x^4y^2 - 14x^2y + 21xy^3$
2) Factor $25xyz^2 - 5x^2y - 15xy^3$

Answers: 1) $7xy(x^3y - 2x + 3y^2)$; 2) $5xy(5z^2 - x - 3y^2)$

14.1.2 Factor by grouping (4 terms)

1. If a GCF of all four terms exists, factor it out.
2. Factor the GCF from the first two terms.
3. Factor the GCF from the last two terms.
4. Factor the common binomial.
5. Double check by multiplying the two binomials to verify that the product is the original polynomial.

Example

Factor $xy + 2x + 2y + 4$ into two binomials.

Answer: $(y + 2)(x + 2)$

Explanation:

<table>
<thead>
<tr>
<th>Step</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. If a GCF of all four terms exists, factor it out.</td>
<td>A GCF of all four terms does not exist.</td>
</tr>
<tr>
<td>2. Factor the GCF from the first two terms.</td>
<td>$x(y + 2) + 2y + 4$</td>
</tr>
<tr>
<td>3. Factor the GCF from the last two terms.</td>
<td>$x(y + 2) + 2(y + 2)$</td>
</tr>
<tr>
<td>4. Factor the common binomial.</td>
<td>$(y + 2)(x + 2)$</td>
</tr>
<tr>
<td>5. Double check by multiplying the two binomials to verify that the product is the original polynomial.</td>
<td>$(y + 2)(x + 2) = xy + 2x + 2y + 4$</td>
</tr>
</tbody>
</table>

Practice problems

1) Factor $4x^2 - 8x - 7xy + 14y$
2) Factor $3x^3 - 6x^2 + 15x - 30$

Answers: 1) $(x - 2)(4x - 7y)$; 2) $3(x^2 + 5)(x - 2)$
14.2 Factoring polynomials

14.2.1 Factor trinomial with leading coefficient of 1

1. If a GCF of all three terms exists, factor it out.
2. Find two factors of $c$ that add up to $b$. ($m$ and $n$).
3. Replace $m$ and $n$ with the factors from step 2: $(x + m)(x + n)$.
4. Double check by multiplying the two binomials to verify that the product is the original trinomial.

Example

Factor $x^2 - 9x + 20$ into two binomials.

Answer: $(x - 4)(x - 5)$

Explanation:

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>If a GCF of all three terms exists, factor it out.</td>
<td>A GCF of all three terms does not exist.</td>
</tr>
<tr>
<td>2</td>
<td>Find two factors of $c$ that add up to $b$. ($m$ and $n$).</td>
<td>Factors of 20 that add up to $-9$ are: $-4$ and $-5$</td>
</tr>
<tr>
<td>3</td>
<td>Replace $m$ and $n$ with the factors from step 2: $(x + m)(x + n)$.</td>
<td>$(x - 4)(x - 5)$</td>
</tr>
<tr>
<td>4</td>
<td>Double check by multiplying the two binomials to verify that the product is the original trinomial.</td>
<td>$(x - 4)(x - 5) = x^2 - 9x + 20$</td>
</tr>
</tbody>
</table>

Practice problems

1) Factor $x^2 + 12x + 27$
2) Factor $x^2 + 4x - 12$

Answers: 1) $(x + 3)(x + 9)$; 2) $(x - 2)(x + 6)$

14.2.2 Factor trinomials with leading coefficient not 1

1. If a GCF of all three terms exists, factor it out.
2. Find two factors of $ac$ that add up to $b$. ($m$ and $n$)
3. Rewrite the expression with the factors from step 2 as: $ax^2 + mx + nx + c$.
4. Factor the four terms by grouping.
5. Double check by multiplying the two binomials to verify that the product is the original trinomial.


**Example**

Factor \(6x^2 - x - 2\) into two binomials.

**Answer:** \((3x - 2)(2x + 1)\)

**Explanation:**

1. If a GCF of all three terms exists, factor it out. | A GCF of all three terms does not exist.
2. Find two factors of ac that add up to b. (m and n). | Factors of \(-12\) that add up to \(-1\) are: \(-4\) and \(3\)
3. Rewrite the expression with the factors from step 2 as: \(ax^2 + mx + nx + c\). | \(6x^2 - 4x + 3x - 2\)
4. Factor the four terms by grouping. | \(6x^2 - 4x + 3x - 2\)
   \[2x(3x - 2) + 1(3x - 2)\]
   \[(3x - 2)(2x + 1)\]
5. Double check by multiplying the two binomials to verify that the product is the original trinomial. | \((3x - 2)(2x + 1) = 6x^2 - x - 2\)

**Practice problems**

1) Factor \(3x^2 - 2x - 8\)
2) Factor \(36x^3 + 33x^2 + 6x\)

**Answers:** 1) \((3x + 4)(x - 2)\); 2) \(3x(4x + 1)(3x + 2)\)

14.2.3 Factor difference of square binomial

<table>
<thead>
<tr>
<th><strong>The expression</strong></th>
<th><strong>factors as:</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>(x^2 - y^2)</td>
<td>((x - y)(x + y))</td>
</tr>
<tr>
<td>(x^2 + y^2)</td>
<td><em>Does not factor; it is prime</em></td>
</tr>
</tbody>
</table>

**Example**

Factor \(5x^2 - 80\) into two binomials, if possible.

**Answer:** \(5(x - 4)(x + 4)\)
Explanation: Once the GCF is factored, the two remaining terms in the binomial are the difference of two squares.

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Factor out the GCF</td>
</tr>
<tr>
<td>2.</td>
<td>Using the shortcut $x^2 - y^2 = (x - y)(x + y)$</td>
</tr>
<tr>
<td>3.</td>
<td>Double check</td>
</tr>
</tbody>
</table>

**Practice problems**

1) Factor $2x^2 - 72$

2) Factor $x^2 - 25$

Answers: 1) $2(x - 6)(x + 6)$; 2) $(x - 5)(x + 5)$

### 14.2.4 Factor perfect square trinomial

<table>
<thead>
<tr>
<th>The expression</th>
<th>factors as:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^2 + 2xy + y^2$</td>
<td>$(x + y)(x + y)$ or $(x + y)^2$</td>
</tr>
<tr>
<td>$x^2 - 2xy + y^2$</td>
<td>$(x - y)(x - y)$ or $(x - y)^2$</td>
</tr>
</tbody>
</table>

**Example**

Factor $9t^2 + 42tu + 49u^2$ into two binomials, if possible.

Answer: $(3t + 7u)^2$

Explanation: The first term and the last term are perfect squares and the middle term is twice the product of the two square roots. This is a perfect square trinomial.

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Factor out the GCF</td>
</tr>
<tr>
<td>2.</td>
<td>Using the shortcut $x^2 + 2xy + y^2 = (x + y)^2$</td>
</tr>
</tbody>
</table>
| 3.   | Double check | $(3t + 7u)(3t + 7u)$  
$= 9t^2 + 42tu + 49u^2$ |
**Practice problems**

1) Does $x^2 - 10x + 25$ fit the pattern of a perfect square trinomial?

2) Does $x^2 + 14x + 40$ fit the pattern of a perfect square trinomial?

Answers: 1) yes; 2) no

**15 Roots and Radicals**

**15.1 Introductions to roots and radicals**

**15.1.1 Find the principal square root**

If $a^2 = b$ and $a \geq 0$, then the **square root** of $b$ is $a$, written as $\sqrt{b} = a$.

If $b$ is a positive number, then $\sqrt{b}$ is the **positive square root** of $b$ and $-\sqrt{b}$ is the **negative square root** of $b$.

If $b$ is a negative number, then $\sqrt{b}$ is **not a real number**.

**Example**

Find the positive square root.

$\sqrt{49}$

Answer: 7, because $7^2 = 49$

**Practice problems**

1) Find $\sqrt{225}$

2) Find $-\sqrt{4}$

Answers: 1) 15; 2) $-2$
15.1.2 Approximate a square root without a calculator

*Example*

Approximate $\sqrt{33}$

Find two perfect squares such that 33 is between them:

\[25 < 33 < 36\]
\[5^2 < 33 < 6^2\]

\[5 < \sqrt{33} < 6.\] Take the square root of each.

The $\sqrt{33}$ lies between 5 and 6.

*Practice problems*

1) Approximate $\sqrt{53}$

2) Approximate $\sqrt{115}$

Answers: 1) between 7 and 8; 2) between 10 and 11

15.1.3 Find a cube root

If $a^n = b$ then the **nth root** of $b$ is $a$, written as $\sqrt[n]{b} = a$.

If $b$ is a negative number and $n$ is an even root, then $\sqrt[n]{b}$ is **not a real number**.

In $\sqrt[n]{b}$, $n$ is the **root**, $b$ is the **radicand**.

*Example*

Find the cube root.

\[\sqrt[3]{125}\]

Answer: 5, because $5^3 = 125$

*Practice problems*

1) Find $\sqrt[3]{8}$

2) Approximate $\sqrt[3]{29}$

Answers: 1) 2; 2) between 3 and 4
15.1.4 Simplify a radical

The **product rule** states $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$

The **quotient rule** states $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, b \neq 0$

To simplify a radical:

1. Complete a factor tree to rewrite the radicand as a product of prime factors. For example, $\sqrt{48} = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}$
2. Use the index to determine how many prime factors to match together. For example, the index of $\sqrt{48}$ is 2, so match 2 of a kind. $\sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot \sqrt{2 \cdot 2 \cdot \sqrt{2 \cdot 2}}}$
3. Simplify. For example, $\sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot \sqrt{2 \cdot 2 \cdot \sqrt{2 \cdot 2}}} = 2 \cdot 2 \cdot \sqrt{2 \cdot 2} = 4\sqrt{2}$

**Example**

Simplify the radical.

$\sqrt{40}$

Answer: $2\sqrt{10}$, because $\sqrt{40} = \sqrt{2 \cdot 2 \cdot 2 \cdot 5} = \sqrt{2 \cdot 2 \cdot \sqrt{2 \cdot 2 \cdot 5}} = 2\sqrt{10}$

Simplify the radical.

$-\sqrt{\frac{75}{81}}$

Answer: $-\frac{5\sqrt{3}}{9}$, because $-\sqrt{\frac{75}{81}} = -\frac{\sqrt{75}}{\sqrt{81}} = -\frac{\sqrt{3 \cdot 5 \cdot 3}}{9} = -\frac{5\sqrt{3}}{9}$

Simplify the radical.

$2\sqrt{4x^7}$

Answer: $4x^3\sqrt{x}$, because $2\sqrt{4x^7} = 2\sqrt{2 \cdot 2 \cdot x \cdot x \cdot x \cdot x \cdot x} = 4x^3\sqrt{x}$

**Practice problems**

1) Simplify $\sqrt{156}$

2) Simplify $\sqrt{x^4y^3}110$

Answers: 1) $2\sqrt{39}$; 2) $x^2y\sqrt{110y}$
15.2 Operations with radicals

15.2.1 Add and subtract radicals

To add or subtract radicals:

1. Simplify each radical.
2. Combine like terms. Like radicals have the same index and radicand.

Example

Add $\sqrt{48} + \sqrt{108}$

Answer:

$$= \sqrt{48} + \sqrt{108}$$
$$= \sqrt{3 \cdot 2 \cdot 2 \cdot 2} + \sqrt{3 \cdot 3 \cdot 2 \cdot 2}$$
$$= 4\sqrt{3} + 6\sqrt{3}$$ Simplify the radicals
$$= 10\sqrt{3}$$ Combine like radicals

Practice problems

1) Add $3\sqrt{12} + \sqrt{27}$
2) Subtract $\sqrt{50} - \sqrt{32}$

Answers: 1) $9\sqrt{3}$; 2) $\sqrt{2}$

15.2.2 Multiply radicals

To multiply radicals:

1. $c^n\sqrt{a} \cdot d^n\sqrt{b} = cd^n\sqrt{ab}$
2. Simplify.

Example

Multiply $(\sqrt{x} + \sqrt{3})(\sqrt{x} - \sqrt{8})$

Answer:

$$= (\sqrt{x} + \sqrt{3})(\sqrt{x} - \sqrt{8})$$
$$= \sqrt{x} \cdot x - \sqrt{x} \cdot 8 + \sqrt{3} \cdot x - \sqrt{3} \cdot 8$$ Use the distributive property (FOIL)
$$= \sqrt{x^2} - 8\sqrt{x} + \sqrt{3x} - \sqrt{24}$$
$$= x - \sqrt{8x} + \sqrt{3x} - 2\sqrt{6}$$ Simplify the radicals
Practice problems

1) Multiply $(\sqrt{2})(\sqrt{18})$

2) Multiply $(\sqrt{51x^4y^5})(\sqrt{9x^5})$

Answers: 1) 6; 2) $3x^4y^2\sqrt{51xy}$

15.2.3 Divide radicals

To divide radicals:

1. $\frac{n\sqrt{a}}{n\sqrt{b}} = \sqrt{\frac{a}{b}}$, $b \neq 0$
2. Simplify.

Example

Divide $\frac{\sqrt{24x^7y^3}}{\sqrt{6x^3y}}$

Answer:

$$= \frac{\sqrt{24x^7y^3}}{\sqrt{6x^3y}}$$

$$= \sqrt{\frac{24x^7y^3}{6x^3y}}$$

Use the quotient rule

$$= \sqrt{4x^4y^2}$$

Simplify the fraction

$$= 2x^2y$$

Simplify the radicals

Practice problems

1) $\frac{\sqrt{b}}{\sqrt{3}}$

2) $\frac{\sqrt{4x^4y^3}}{\sqrt{2xy^2}}$

Answers: 1) $\sqrt{2}$; 2) $x\sqrt{2xy}$
15.2.4 Rationalize the denominator

The process rationalizing the denominator is used to rewrite a fraction radical expression without a radical in the denominator.

1. Begin with the radical in this form: \( \frac{\sqrt{a}}{\sqrt{b}} \), \( b \neq 0 \)
2. Multiply the fraction by 1 in the form of \( \frac{\sqrt{b}}{\sqrt{b}} \)
3. Simplify the radicals in the numerator and denominator.

If the denominator contains a radical expression, the steps to rationalizing the denominator are:

1. Begin with the radical in this form: \( \frac{a}{c+\sqrt{b}} \), where \( a \) can be any non-zero number, variable, or expression.
2. Multiply the fraction by 1 in the form of \( \frac{c-\sqrt{b}}{c-\sqrt{b}} \)
3. Simplify the radicals in the numerator and denominator.

**Example**

Example 1: Rationalize the denominator \( \frac{23x}{\sqrt{3x}} \)

Answer:

\[
\frac{23x}{\sqrt{3x}} \cdot \frac{\sqrt{3x}}{\sqrt{3x}} = \frac{23x\sqrt{3x}}{\sqrt{9x^2}} = \frac{23x\sqrt{3x}}{3x} \cdot \frac{\sqrt{3x}}{\sqrt{3x}} = \frac{23\sqrt{3x}}{3} \cdot \frac{\sqrt{3x}}{\sqrt{3x}} = \frac{23\sqrt{3x}}{3} \]

Multiply the fraction by 1 in the form \( \frac{\sqrt{3x}}{\sqrt{3x}} \)

Simplify the radicals

Simplify the fraction

Example 2: Rationalize the denominator \( \frac{3}{\sqrt{3}+1} \)

Answer:

\[
\frac{3}{\sqrt{3}+1} = \frac{3\sqrt{3}-1}{(\sqrt{3}+1)(\sqrt{3}-1)} \cdot \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{3\sqrt{3}-1}{\sqrt{3}+\sqrt{3}-\sqrt{3}-1} \cdot \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{3\sqrt{3}-1}{2} \cdot \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{3\sqrt{3}-1}{2} \]

Multiply the fraction by 1 in the form \( \frac{\sqrt{3}-1}{\sqrt{3}-1} \)

Multiply the radicals

Simplify the fraction
Practice problems

1) Rationalize the denominator $\frac{1}{2-\sqrt{3}}$

2) Rationalize the denominator $\frac{\sqrt{3}}{1+\sqrt{5}}$

Answers: 1) $2 + \sqrt{3}$, 2) $\frac{\sqrt{3} - \sqrt{15}}{-4}$

15.3 Radical equations

The **squaring property of equality** states *if* $a = b$ *then* $a^2 = b^2$

To **solve** an equation containing radicals:

1. Isolate a radical on one side of the equation.
2. Square both sides of the equation.
3. Simplify both sides of the equation.
4. If the equation still contains a radical, repeat steps 1-3.
5. Solve.
6. Check for and discard extraneous solutions. Extraneous solutions will not check in the original equation.

15.3.1 Solve a radical equation

**Example**

$$\sqrt{x} + 5 = 7$$

Answer: $x = 44$

Explanation:

$$\sqrt{x} + 5 = 7$$
$$\left(\sqrt{x} + 5\right)^2 = 7^2$$
$$x + 5 = 49$$
$$x = 44$$

Check

$$\sqrt{44 + 5} = 7$$
$$\sqrt{49} = 7$$

Practice problems

1) Solve $\sqrt{2x - 2} = x - 1$

2) Solve $\sqrt{x + 1} = 2$

Answers: 1) $x = 1$ and $x = 3$; 2) $x = 3$
15.3.2 Solve and check for extraneous solutions

Example

\[ \sqrt{-7x + 56} = x - 8 \]

Answer: \( x = 8 \)

Explanation:
\[ \sqrt{-7x + 56} = x - 8 \quad \text{Isolate a radical} \]
\[ (\sqrt{-7x + 56})^2 = (x - 8)^2 \quad \text{Square both sides} \]
\[ -7x + 56 = x^2 - 16x + 64 \quad \text{Simplify both sides} \]
\[ 0 = x^2 - 9x + 8 \quad \text{Solve} \]
\[ 0 = (x - 8)(x - 1) \]
\[ x = 8 \text{ and } x = 1 \]

\[ \sqrt{-7(8) + 56} = 8 - 8 \quad \text{Check } x = 8 \]
\[ \sqrt{0} = 0 \]

\[ \sqrt{-7(1) + 56} = 1 - 8 \quad \text{Check } x = 1 \]
\[ \sqrt{49} \neq -7 \quad \text{This is not a solution} \]

Practice problems

1) Solve \( \sqrt{x + 4} = x - 2 \)
2) Solve \( \sqrt{x + 2} = x \)

Answers: 1) \( x = 5 \); 2) \( x = 2 \)

16 Quadratic Equations

16.1 Solving

16.1.1 Solve by factoring

The Zero Product Property states that if \( A \cdot B = 0 \), then \( A = 0 \) or \( B = 0 \). Therefore, if the product of two or more factors equals zero, then the solutions to the equation are found when each factor equals zero.

1. Put the quadratic equation in the form \( ax^2 + bx + c = 0 \)
2. Factor \( ax^2 + bx + c \)
3. Set each factor equal to 0 and solve.
Quadratic equations will have two, one, or no solutions.

**Example**

Solve $x^2 - 17x + 52 = 0$

Answers: $x = 13$ and $x = 4$

Explanation: This equation is already in standard form equal to 0. Factor the trinomial. Using the Zero Product Property, set each factor equal to 0 and solve.

$$x^2 - 17x + 52 = 0$$

$$(x - 13)(x - 4) = 0$$

$x - 13 = 0$  \quad $x - 4 = 0$

$x = 13$  \quad $x = 4$

**Practice problems**

1) Solve $x^2 + 2x - 24 = 0$

2) Solve $y^2 - 8y + 15 = 0$

Answers: 1) $x = -6$ and $x = 4$; 2) $y = 3$ and $y = 5$

16.1.2 Solve using the square root property

The **square root property** states if $x^2 = k$, assuming $k \geq 0$, then $x = \pm\sqrt{k}$

To solve a quadratic equation in the form $x^2 = k$ or $(x + c)^2 = k$

1. Take the square root of both sides.
2. Solve.

**Example**

$$(x - 5)^2 = 16$$

Answer: $x = 9$ and $x = 1$

Explanation:

$$(x - 5)^2 = 16$$  \quad Isolate $(x - 5)^2$

$$\sqrt{(x - 5)^2} = \sqrt{16}$$  \quad Take the square root of both sides

$$x - 5 = \pm 4$$  \quad Simplify

Solve: $x - 5 = 4$  \quad And solve: $x - 5 = -4$

$x = 9$  \quad $x = 1$
Practice problems

1) Solve \((x - 1)^2 = 4\)

2) Solve \(2x^2 + 3 = 75\)

Answers: 1) \(x = 3 \text{ and } x = -1\); 2) \(x = 6 \text{ and } x = -6\)

16.1.3 Solve by completing the square

To solve a quadratic equation by completing the square:

1. If \(a \neq 1\), divide both sides of the equation by \(a\) and set equal to 0. The quadratic is now in the form \(x^2 + \frac{b}{a}x + \frac{c}{a} = 0\).
2. Subtract by \(\frac{c}{a}\) from both sides.
3. Find \(m = \left(\frac{b}{a}\right)^2\)
4. Add \(m\) to both sides of the equation. \(x^2 + \frac{b}{a}x + \frac{c}{a} = 0\)
5. Factor \(x^2 + \frac{b}{a}x + m\). It will factor as a perfect square trinomial into \(\left(x + \frac{b}{2a}\right)^2\)
6. Using the Square Root Property, solve.

Example

\[x^2 + 4x + 2 = 0\]

Answer: \(x = -2 + \sqrt{2} \text{ and } x = -2 - \sqrt{2}\)

Explanation:
\[x^2 + 4x + 2 = 0\] The quadratic is in the form \(x^2 + bx + c = 0\)
\[x^2 + 4x = -2\] Subtract \(c\) from both sides
\[x^2 + 4x + 4 = -2 + 4\] Add \(\left(\frac{b}{2}\right)^2\) to both sides
\[(x + 2)^2 = 2\] Factor and simplify
\[\sqrt{(x + 2)^2} = \sqrt{2}\] Solve using the square root property
\[x + 2 = \pm \sqrt{2}\] And solve
\[x + 2 = \sqrt{2}\]
\[x = -2 + \sqrt{2}\]
\[x + 2 = -\sqrt{2}\]
\[x = -2 - \sqrt{2}\]
**Practice problems**

1) What number can you add to $x^2 - 8x$ to complete the perfect square trinomial?

2) Complete the square to solve $x^2 + 10x - 75 = 0$

**Answers:** 1) 16; 2) $x = 5$ and $x = -15$

16.1.4 Solve using the quadratic formula

To **solve** a quadratic equation in the form $ax^2 + bx + c = 0$ use the following formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Example**

$3x^2 + 6x - 7 = 0$

Answer: $x = \frac{-3 \pm \sqrt{30}}{3}$

**Explanation:**

$3x^2 + 6x - 7 = 0$ \hspace{1cm} \text{The equation is in form } ax^2 + bx + c = 0

$a = 3, b = 6, c = -7$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ \hspace{1cm} \text{The quadratic formula}

$x = \frac{-6 \pm \sqrt{36 - 4(3)(-7)}}{2(3)}$ \hspace{1cm} \text{Replace } a = 3, b = 6, c = -7

$x = \frac{-6 \pm \sqrt{120}}{6}$ \hspace{1cm} \text{Simplify}

$x = \frac{-6 \pm 2\sqrt{30}}{6}$ \hspace{1cm} \text{Simplify the radical}

$x = \frac{-3 \pm \sqrt{30}}{3}$ \hspace{1cm} \text{Simplify the fraction}

**Practice problems**

1) Solve $x^2 - 8x + 13 = 0$

2) Solve $x^2 + 3x - 4 = 0$

**Answers:** 1) $x = 4 \pm \sqrt{3}$; 2) $x = 1$ and $x = -4$
17 Rational Expressions

17.1 Introduction to rational expressions

A rational expression is a fraction with polynomials in the numerator and the denominator.

To evaluate a rational expression, replace the variable with its assigned numerical value and then simplify.

A rational expression is undefined for those values that make the denominator equal to 0.

To simplify a rational expression, factor completely the numerator and denominator. Factors in common in both the numerator and denominator can be canceled, or crossed-out.

17.1.1 Find values that make a rational expression undefined

Example

Find any numbers for which each rational expression is undefined.

\[
\frac{3x - 12}{x - 4}
\]

Answer: 4

Explanation: Solve \(x - 4 = 0\). The denominator is 0 when \(x = 4\).

Practice problems

1) Find values that make the rational expression \(\frac{x - 1}{3x + 1}\) undefined

2) Find values that make the rational expression \(\frac{x}{x - 3}\) undefined

Answers: 1) \(x = -\frac{1}{3}\); 2) \(x = 3\)

17.1.2 Simplify rational expressions

Example

Simplify:

\[
\frac{x^2 + x - 12}{x^2 + 4x}
\]

Answer: \(\frac{x - 3}{x}\)
Explanation: In factored form, the rational expression is \( \frac{(x + 4)(x - 3)}{x(x + 4)} \). Cross out the matching factors \( \frac{(x + 4)(x - 3)}{x(x + 4)} \). The simplified fraction is \( \frac{x - 3}{x} \).

**Practice problems**

1) Simplify \( \frac{2x^2 - 2x}{2x^3 - 4x^2 + 2x} \)

2) Simplify \( \frac{x^2 + 6x + 9}{x^2 - 9} \)

Answers: 1) \( \frac{1}{x - 1} \); 2) \( \frac{x + 3}{x - 3} \)

**17.2 Operations with rational expressions**

**17.2.1 Multiply rational expressions**

To **multiply** rational expressions:

1. Factor completely the numerator and the denominator.
2. Write as one fraction, keeping the numerator and denominator in factored form.
3. Cross out factors in common in both the numerator and denominator.

**Example**

\[
\frac{10a + 90}{a^2 - 11a + 30} \cdot \frac{a^2 - 13a + 40}{a^2 + a - 72}
\]

Answer: \( \frac{10}{a - 6} \)

Explanation: In factored form: \( \frac{10(a + 9)}{(a - 6)(a - 5)} \cdot \frac{(a - 5)(a - 8)}{(a + 9)(a - 8)} \)

Cross out matching factors: \( \frac{10}{(a - 6)(a - 5)} \cdot \frac{(a - 5)(a - 8)}{(a + 9)(a - 8)} \)

Simplify: \( \frac{10}{a - 6} \)
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Practice problems

1) \( \frac{2}{3x^2} \cdot \frac{12x^3}{14} \)
2) \( \frac{6x^2}{x^2+x-12} \cdot \frac{x+4}{20x} \)

Answers: 1) \( \frac{4x}{7} \); 2) \( \frac{3x}{10(x-3)} \)

17.2.2 Divide rational expressions

To divide rational expressions, turn it into multiplication by multiplying the first fraction by the reciprocal of the second fraction. Then, follow the steps above.

Example

\[
\frac{v + 1}{v - 5} \div \frac{1}{v^2 - 15v + 50}
\]

Answer: \((v + 1)(v - 10)\)

Explanation: In factored form: \(\frac{(v+1)}{(v-5)} \div \frac{1}{(v-5)(v-10)}\)

Multiply the first fraction by the reciprocal of the second fraction: \(\frac{(v+1)}{(v-5)} \cdot \frac{(v-5)(v-10)}{1}\)

Cross out matching factors: \(\frac{(v+1)}{(v-5)} \cdot \frac{(v-5)(v-10)}{1}\)

Simplify: \((v + 1)(v - 10)\)

Practice problems

1) \( \frac{x^2+7x+10}{2x^2-11x+15} \div \frac{x^2+3x+2}{2x^2-3x-5} \)
2) \( \frac{y^2+3y}{y^2-9} \div \frac{y^2-4y}{y^2+y-20} \)

Answers: 1) \( \frac{x+5}{x-3} \); 2) \( \frac{y+5}{y-3} \)

17.2.3 Add rational expressions

To add or subtract rational expressions with the same denominator:

1. Add or subtract the numerators only. The denominator remains the same.
2. Factor completely the numerator and the denominator.
3. Cross out factors in common in both the numerator and denominator.
To add or subtract rational expressions with different denominators, the fractions must be rewritten with the least common denominator (LCD):

1. Factor completely the denominators.
2. Find the LCD. The LCD consists of each unique factor.
3. Create equivalent fractions with the LCD in the denominator.
4. Perform the required operations in the numerator only. The denominator remains the same.
5. Factor completely the numerator.
6. Simplify by crossing out factors in common in both the numerator and denominator.

**Example**

\[
\frac{2n}{4} + \frac{7n + 6}{20n - 8}
\]

Answer: \(\frac{10n^2 + 3n + 6}{4(5n - 2)}\)

Explanation:

Factor each denominator: \(\frac{2n}{4} + \frac{7n + 6}{4(5n - 2)}\)

The LCD of 4 and 4(5n − 2) is 4(5n − 2)

Create equivalent fractions with the LCD as the denominator \(\frac{2n(5n - 2)}{4(5n - 2)} + \frac{7n + 6}{4(5n - 2)}\)

Simplify and add the numerator \(\frac{10n^2 - 4n}{4(5n - 2)} + \frac{7n + 6}{4(5n - 2)} = \frac{10n^2 - 4n + 7n + 6}{4(5n - 2)} = \frac{10n^2 + 3n + 6}{4(5n - 2)}\)

The numerator cannot be factored; the final answer is \(\frac{10n^2 + 3n + 6}{4(5n - 2)}\)

**Practice problems**

1. \(\frac{6x}{7x^2y} + \frac{5y^3}{3xy^4}\)

2. \(\frac{1}{x+5} + \frac{4x}{x^2-25}\)

Answers: 1) \(\frac{53}{21xy}\); 2) \(\frac{5(x-1)}{(x+5)(x-5)}\)
17.2.4 Subtract rational Expressions

**Example**

\[
\frac{x + 1}{x - 4} - \frac{x + 1}{x^2 - 7x + 12}
\]

Answer: \(\frac{x+1}{x-3}\)

Explanation:

Factor each denominator:

\[
\frac{x+1}{(x-4)} = \frac{x+1}{(x-4)(x-3)}
\]

The LCD of \((x - 4)\) and \((x - 4)(x - 3)\) is \((x - 4)(x - 3)\)

Create equivalent fractions with the LCD as the denominator:

\[
\frac{(x+1)(x-3)}{(x-4)(x-3)} - \frac{(x+1)}{(x-4)(x-3)} = \frac{x^2 - 2x - 3}{(x-4)(x-3)} = \frac{x^2 - 3x - 4}{(x-4)(x-3)}
\]

Simplify and subtract the numerator:

\[
\frac{x^2 - 2x - 3}{(x-4)(x-3)} = \frac{(x-4)(x+1)}{(x-4)(x-3)} = \frac{x+1}{x-3}
\]

**Practice problems**

1) \(\frac{1}{x+5} - \frac{2}{2x-10}\)

2) \(\frac{7x}{x^2y} - \frac{3}{2xy^3}\)

Answers: 1) \(\frac{-10}{(x-5)(x+5)}\); 2) \(\frac{14y^2-3}{2xy^3}\)

17.2.5 Simplify a complex rational expression

**Example**

Simplify the complex fraction:

\[
\frac{x-8}{x^2-4} \div \frac{x^2-64}{x-2}
\]

Answer: \(\frac{1}{(x+2)(x+8)}\)

Explanation: First rewrite the complex fraction as a division problem:
\[
\frac{x - 8}{x^2 - 4} \div \frac{x^2 - 64}{x - 2}
\]

Next, rewrite the division as multiplication by the reciprocal of the divisor:

\[
\frac{x - 8}{x^2 - 4} \cdot \frac{x - 2}{x^2 - 64}
\]

Factor the numerators and denominators:

\[
\frac{(x - 8)}{(x - 2)(x + 2)} \cdot \frac{(x - 2)}{(x - 8)(x + 8)}
\]

Use the fact that any number divided by itself equals 1 to simplify:

\[
\frac{1}{(x + 2)(x + 8)}
\]

Practice problems

1)\[
\begin{align*}
\frac{1}{10} + \frac{3}{2} &= \frac{5}{2} \\
\frac{10}{5} + \frac{2}{5} &= \frac{12}{5}
\end{align*}
\]

2)\[
\begin{align*}
\frac{x - 5}{x^2 - 100} &= \frac{x^2 - 25}{2x + 20} \\
\frac{x - 5}{(x - 10)(x + 5)} &= \frac{x + 5}{2(x + 10)}
\end{align*}
\]

Answers: 1) \(\frac{2}{3}\); 2) \(\frac{2}{(x - 10)(x + 5)}\)

18 Rational Equations

18.1 Solving rational equations

To solve an equation containing rational expressions,

Step 1: Get rid of the fractions.

Method 1: For equations that are, or can easily be written as, a fraction equal to a fraction, the cross-multiplication shortcut is a good first step towards solving the equation.

\[
\frac{a}{b} = \frac{c}{d}, \text{ where } b \neq 0 \text{ and } d \neq 0, \text{ is equivalent to } ad = bc
\]
**Method 2**: Multiply each side of the equation by the **LCD** to get rid of the fractions.

1. Find the **LCD** of the rational expressions in the equation.
2. Multiply both sides of the equation by the **LCD**.
3. Simplify each term.

**Step 2: Solve the equation.**

**Step 3**: Check for and discard **excluded values**. An excluded value is one that will make a rational expression undefined, in other words, when the denominator is equal to 0.

### 18.1.1 Solve rational equations (Method 1)

**Example**

\[
\frac{1}{v} + \frac{3v + 12}{v^2 - 5v} = \frac{7v - 56}{v^2 - 5v}
\]

**Answer**: 21

**Explanation**: Solving using **Method 1**: Notice the two fractions with a common denominator.

\[
\frac{1}{v} + \frac{3v + 12}{v^2 - 5v} = \frac{7v - 56}{v^2 - 5v} \\
\frac{1}{v} = \frac{7v - 56}{v^2 - 5v} - \frac{3v + 12}{v^2 - 5v} \\
\frac{1}{v} = \frac{7v - 56 - 3v - 12}{v^2 - 5v} \\
\frac{1}{v} = \frac{4v - 68}{v^2 - 5v} \\
v^2 - 5v = v(4v - 68) \quad \text{Using the cross-multiplication short cut} \\
v^2 - 5v = 4v^2 - 68v \\
0 = 3v^2 - 63v \\
0 = 3v(v - 21) \\
3v = 0 \text{ or } v - 21 = 0 \\
v = 0 \text{ or } 21
\]

The excluded values are 0 and 5, so the final answer is \(v = 21\).
**Practice problems**

Solve:

1) \[ \frac{x^2-1}{x-3} = \frac{8}{x-3} \]

2) \[ \frac{x-1}{x^2-4} = \frac{1}{4} \]

**Answers:** 1) \( x = -3 \); 2) \( x = 0 \) or 4

---

**18.1.2 Solve rational equations (Method 2)**

**Example**

\[ \frac{1}{x} = \frac{6}{5x} + 1 \]

Answer: \( x = -\frac{1}{5} \)

Explanation: Solving using **Method 2: Clear the fractions** by multiplying both sides by LCD of \( 5x \).

\[
5x \left( \frac{1}{x} \right) = \left( \frac{6}{5x} + 1 \right) 5x
\]

\[
\frac{5x}{x} = \left( \frac{30x}{5x} + 5x \right)
\]

\[ 5 = 6 + 5x \]

\[ -1 = 5x \]

\[ -\frac{1}{5} = x \]

The excluded value is 0.

**Practice problems**

1) \[ \frac{x}{5} - 3 = \frac{2}{5} \]

2) \[ \frac{x+1}{14} = \frac{2}{7} \]

**Answers:** 1) \( x = 17 \); 2) \( x = 3 \)
19 Functions

19.1 Introduction to Functions

Definitions:

**Domain**: The domain of a function is the set of all possible values for the input \((x)\) of the function.

**Range**: The range of a function is the set of all possible values for the output \((y)\) of the function.

19.1.1 Determine whether a relation is a function

*Example*

Is the relation \[\{(-4,7), (-2,5), (-1,3), (2,5), (4, -6)\}\] a function?

Solution: A function is a relationship between two sets of numbers. For each input, or number in the first set, there is exactly one output, or number, in the second set.

This relationship (or relation) can be shown using ordered pairs.

Each ordered pair consists of two numbers, an input and an output. In this example, all of the inputs are different and each input has only one output. So this relation is a function.

The domain is the set \([-4, -2, -1, 2, 4]\). The range is the set \([-6, 3, 5, 7]\).

*Practice problems*

1) Is the relation \[\{(-5,2), (-3,3), (0,6), (0,4), (2,1)\}\] a function?

2) Is the relation \[(1,2), (3,2), (5,1), (0,0)\]\ a function?

Answers: 1) no; 2) yes
19.1.2 Determine if a graph is a function (vertical line test)

**Vertical Line Test**
A graph represents a function if there are no vertical lines that intersect the graph at more than one point.

![Graphs](image)

- **Is a Function**
  - No vertical line will cross the graph more than once.

- **NOT a Function**
  - There is a vertical line that crosses the graph more than once.

**Example**

Determine if this graph is a function

![Graph](image)

Because this graph fails the vertical line test, it is not a function.

**Practice problems**

Is graph #1 or graph #2 a function?

1) 

![Graph](image)

2) 

![Graph](image)

Answers: Graph #1 is a function
19.2 Function Notation

19.2.1 Evaluate an expression in function notation

*Example*

Given \( f(x) = -6x + 4 \), find \( f(2) \)

Answer: \( f(2) = -8 \)

Explanation: Substitute 2 for \( x \) in the function and simplify

\[ f(2) = -6(2) + 4 = -8 \]

*Practice problems*

1) Evaluate \( f(x) = x^3 - 2x^2 + 4 \) at \( x = -1 \)

2) Given \( g(x) = x + 7 \) find \( g(b) \)

Answers: 1) 1; 2) \( b + 7 \)

20 Graphing Functions

20.1 Linear functions

To graph a linear function in the form \( f(x) = mx + b \).

1. Plot the y-intercept which is the point \((0, b)\)
2. Plot a second point using the slope \( m \) to rise and run from the y-intercept.

20.1.1 Graph a linear function and find domain and range

*Example*

Graph \( f(x) = 2x + 1 \), find the domain and range
Answer:

\[ y = 2x + 1 \]

Explanation: Begin by plotting the \( y \)-intercept at the point \((0,1)\). The slope of the line is 2, so as a fraction \( \frac{2}{1} \), the rise is 2 and the run is 1. Plot a second point at \((1,3)\). Connect the dots.

The domain of the linear functions is all real numbers, because \( x \) can take any value. The range of the linear functions is all real numbers, because \( y \) can take any value.

**Practice problems**

1) Graph \( f(x) = x - 4 \), find the domain and range

2) Graph \( f(x) = -x + 5 \), find the domain and the range

Answers:

1) The domain and range are all real numbers.

\[ y = x - 4 \]

2) The domain and range are all real numbers.

\[ y = -x + 5 \]
20.2 Quadratic Functions

20.2.1 Graph a quadratic function (vertex form) finding vertex and y-intercept

To graph a quadratic function in the form \( f(x) = a(x - h)^2 + k \), find the vertex and at least one other point.

1. Find and plot the vertex \((h, k)\)
2. Plot the y-intercept which is the point \((0, f(0))\)
3. If \(a > 1\), then the parabola opens up. If \(a < 1\), then the parabola opens down.
4. If necessary, plot the x-intercept(s) which are the solution(s) when \(a(x - h)^2 + k = 0\)

Example

Graph \( f(x) = (x - 1)^2 + 2 \)

Answer: \( y = (x - 1)^2 + 2 \)

Explanation: Parabola in the vertex form is \( f(x) = a(x - h)^2 + k \). The vertex is at the point \((h, k)\). To find the y-intercept, set \(x = 0\).

In this example \(h = 1, k = 2\), so the vertex is the point \((1, 2)\). Set \(x = 0\) to find the y-intercept

\[
f(0) = (0 - 1)^2 + 2 = 1 + 2 = 3
\]

Since \(a > 0\), the parabola opens up.

Practice problems

1) Find the y-intercept, vertex, and direction of \( f(x) = -(x + 2)^2 - 1 \)

2) Graph \( f(x) = 2(x + 2)^2 + 1 \)

Answers: 1) \( y – intercept = (0, -5), \ vertex = (-2, -1), \ opens \ down; \)
2) \( y - \text{intercept} = (0,9), vertex = (-2,1), \text{opens up} \)

\[
y = 2(x + 2)^2 + 1
\]

20.2.2 Graph a quadratic function (standard form) finding vertex and \( y \)-intercept

To graph a quadratic function in the form \( y = ax^2 + bx + c \), find the vertex and at least one other point.

1. Find and plot the vertex \((x, y)\):
   a. Use the Vertex Formula \( x = \frac{-b}{2a} \) to find the \( x \) value of the vertex.
   b. Plug in the \( x \) found from the vertex formula into \( y = ax^2 + bx + c \) and solve for \( y \).
2. Plot the \( y \)-intercept which is the point \((0, c)\)
3. If \( a > 1 \), then the parabola opens up. If \( a < 1 \), then the parabola opens down.
4. If necessary, plot the \( x \)-intercept(s) which are the solution(s) when \( ax^2 + bx + c = 0 \)

Example

Graph \( y = x^2 + 6x - 7 \)

Answer: \( y = x^2 + 6x - 7 \)
Explanation:
Find the vertex using the vertex formula $x = -\frac{b}{2a}$

$x = -\frac{6}{2} = -3$  
The $x$ value of the vertex is $-3$.

Plug the $x$ value of the vertex into the equation to find the $y$ value of the vertex.

$y = (-3)^2 + 6(-3) - 7$
$y = 9 - 18 - 7 = -16$
The vertex is $(-3, -16)$

Find the $y$-intercept $(0, c) = (0, -7)$

Find the $x$-intercept(s) by setting $y = 0$ and solving for $x$

$0 = x^2 + 6x - 7$
$0 = (x + 7)(x - 1)$
$x + 7 = 0$ and $x - 1 = 0$
$x = -7$ and $x = 1$

The $x$-intercepts are $(-7,0)$ and $(1,0)$

**Practice problems**

1) Find the vertex, $y$-intercept and direction of $f(x) = -x^2 + 6x - 7$

2) Find the vertex, $y$-intercept, $x$-intercepts, and the direction of $f(x) = x^2 - 8x + 15$

**Answers:**

1) vertex = $(3, 2)$, $y$ - intercept = $(0, -7)$, opens down;

2) vertex = $(4, -1)$, $y$ - intercept = $(0, 15)$, $x$ - intercepts = $(5, 0), (3, 0)$, opens up

**20.2.3 Find the domain and range of a quadratic function**

To determine the domain, or all possible $x$-values, of the quadratic function, look for the left-most and right-most $x$-values on the graph. To determine the range, or all possible $y$-values, of the quadratic function, look for the lower-most and upper-most $y$-values on the graph.
Example

Find the domain and range of \( f(x) = x^2 - 4x + 3 \)

Solution: The domain of the quadratic functions is all real numbers. The minimum \( y \)-value of this function is \(-1\) and the graph continues to extend upward, so the range of this function is \( y \geq -1 \). The graph of this function is:

\[
y = x^2 - 4x + 3
\]

Practice problems

1) Find the domain and range of \( f(x) = -x^2 + 6 \)

2) Find the domain and range of \( f(x) = x^2 + 1 \)

Answers: 1) domain is all real numbers, range is \( y \leq 6 \); 2) domain is all real numbers, range is \( y \geq 1 \).
20.3 Square root functions

20.3.1 Graph a square root function

Example

Graph \( f(x) = \sqrt{2x} - 1 \)

\[
y = \sqrt{2x} - 1
\]

Solution: Make a table.

Choose several values for \( x \) and evaluate the function for each of these values:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \sqrt{2x} - 1 )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 - 1 = -1</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>2 - 1 = 1</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>4 - 1 = 3</td>
<td>3</td>
</tr>
<tr>
<td>18</td>
<td>6 - 1 = 5</td>
<td>5</td>
</tr>
</tbody>
</table>

Each row in the table forms an ordered pair. Plot these ordered pairs as points.
Practice problems

1) Graph \( f(x) = \frac{1}{2}x + 2 \)

2) Graph \( f(x) = \sqrt{x} + 3 \)

Answers: 1) \( y = \frac{1}{2}x + 2 \)

\[ y = \sqrt{x} + 3 \]

20.3.2 Find the domain and range of a square root function

\[ y = \sqrt{x} \]
\[ y = \sqrt{x - 1} \]
\[ y = \sqrt{x - 1} + 2 \]

Example

Find the domain and the range of this function \( f(x) = \sqrt{2x + 16} \)

Answer: The domain is all values of \( x \) such that \( x \geq -8 \) and the range is all values of \( y \) such that \( y \geq 0 \).
Explanation: Because you are working with only real numbers, you cannot take the square root of a negative number. So the value of the radicand cannot be negative. Therefore:

\[ 2x + 16 \geq 0 \]

Now solve this inequality for \( x \) to find the domain of the function. Subtract 16 from both sides

\[ 2x \geq -16 \]

Divide by 2:

\[ x \geq -8 \]

Note that the square root symbol means to find the positive square root. So:

\[ y \geq 0. \]

**Practice problems**

1) Find the domain and range of \( f(x) = 3 - \sqrt{x} \)

2) Find the domain and range of \( f(x) = \sqrt{x^2 - 9} \)

**Answers:**

1) Domain: \( x \geq 0 \), Range: \( y \leq 3 \),
2) Domain: \( x \geq 3 \) or \( x \leq -3 \), Range: \( y \geq 0 \)

### 20.4 Exponential functions

#### 20.4.1 Graph an exponential function

To create the exponential graph \( f(x) = 2^x \) above, make a table of values and plot the points:

- \((0, 1)\)
- \((-3, \frac{1}{8})\)
- \((-2, \frac{1}{4})\)
- \((-1, \frac{1}{2})\)
- \((1, 2)\)
- \((2, 4)\)
- \((3, 8)\)

*The x-axis is an asymptote.*

**Example**

To create the exponential graph \( f(x) = 2^x \) above, make a table of values and plot the points:
<table>
<thead>
<tr>
<th>$x$</th>
<th>$2^x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>$2^{-3} = \frac{1}{8}$</td>
</tr>
<tr>
<td>-2</td>
<td>$2^{-2} = \frac{1}{4}$</td>
</tr>
<tr>
<td>-1</td>
<td>$2^{-1} = \frac{1}{2}$</td>
</tr>
<tr>
<td>0</td>
<td>$2^0 = 1$</td>
</tr>
<tr>
<td>1</td>
<td>$2^1 = 2$</td>
</tr>
<tr>
<td>2</td>
<td>$2^2 = 4$</td>
</tr>
<tr>
<td>3</td>
<td>$2^3 = 8$</td>
</tr>
</tbody>
</table>

**Practice problems**

1) Graph $f(x) = \left(\frac{1}{2}\right)^x$

2) Graph $f(x) = 2^x + 1$

**Answers:**

1) $y = \left(\frac{1}{2}\right)^x$

2) $y = 2^x + 1$
20.4.2 Find the domain and range of an exponential function

Example

Find the domain and range of \( f(x) = 2^x \)

Explanation: The function \( f(x) = 2^x \) can have any input value, so the domain is all real numbers. The output values approach 0, but never reach 0, so the range is \( y > 0 \).

Practice problems

1) Find the domain and range for \( y = \left( \frac{1}{2} \right)^x \)

2) Find the domain and range for \( y = 2^x - 1 \)

Answers: 1) domain is all real numbers, range is \( y > 0 \); 2) domain is all real numbers, range is \( y > -1 \)
21 Math Placement Practice Test 2

1. An elephant weighs 1,495 kilograms. If this number was rounded to the nearest hundred, how many zeros would it have?
   A. one
   B. three
   C. none
   D. two

2. Round 1,295.957 to the nearest tenth.
   A. 1,295.9
   B. 1,295.96
   C. 1,296.0
   D. 1,300

3. A football field is in the shape of a rectangle that measures about 120 yards long and 53 yards wide. What is the distance around a football field?
   A. 346 yards
   B. 253 yards
   C. 173 yards
   D. 553 yards

4. Iny had 1,308 songs on her playlist. She deletes 51 of them. How many songs are left?
   A. 1,259
   B. 257
   C. 1,257
   D. 1,357

5. Malcolm is driving 1,373 miles from Wichita to Charleston for a family reunion. He drives 468 miles the first day and 434 miles the second day. Round each distance to the nearest ten and estimate about how many miles Malcolm has left to drive.
   A. 400 miles
   B. 500 miles
   C. 470 miles
   D. 480 miles

6. Round each measurement to the nearest ten. Then estimate the area of the rectangle.

![Rectangle diagram]

A. 3200 square inches
B. 260 square inches
C. 4000 square inches
D. 4500 square inches
7. Micah is twice as old as Richard. Richard is three times as old as Ken. Ken is six years old. How old is Micah?
   A. 8
   B. 18
   C. 11
   D. 36

8. Which of the following is the prime factored form of the lowest common denominator of \(\frac{15}{16} + \frac{5}{12}\)?
   A. \(2 \cdot 4 \cdot 6\)
   B. \(2 \cdot 2 \cdot 2 \cdot 3\)
   C. \(2 \cdot 2 \cdot 2 \cdot 2 \cdot 3\)
   D. \(4 \cdot 4 \cdot 3\)

9. Michael is planning a road trip. He estimates that he will drive a total of \(9\frac{3}{4}\) hours. He wants to stop for gas after \(\frac{1}{3}\) of the total time has passed. How long will he drive before he stops for gas? Express your answer as a mixed number.
   A. \(2\frac{1}{4}\) hours
   B. \(3\frac{1}{4}\) hours
   C. \(13\frac{1}{4}\) hours
   D. \(9\frac{1}{4}\) hours

10. Rosa has \(3\frac{3}{4}\) pounds of dough. She uses \(\frac{1}{8}\) of a pound for one roll. How many rolls could be made from Rosa’s dough?
    A. \(3\frac{1}{6}\)
    B. 9
    C. 30
    D. 18

11. Find the value of the expression when \(x = 4\) and \(y = 7\).
    \((2x - 12) + \frac{1}{2}xy - 10\)
    A. 6
    B. 8
    C. 0
    D. \(-16\)

12. What is the value of \(|-6| - |6| - (-6)|\)?

    The solution is __________.
13. Match each number of the left with the correct description on the right. Answer options on the right may be used more than once.
   A. \(\pi\)  
   B. \(-15\)  
   C. \(-0.314314 \ldots\)  
   D. \(\frac{3}{4}\)  
   E. \(-0.\overline{3}\)  
   F. This is an integer  
   G. This is a rational number, but not an integer  
   H. This is an irrational number

14. Evaluate the expression \((-3.8) + 2 + y\) for the given values of \(y\).

<table>
<thead>
<tr>
<th>(y) value</th>
<th>Value of expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-3.8</td>
</tr>
<tr>
<td>1.8</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>3.8</td>
<td></td>
</tr>
</tbody>
</table>

15. Find \(-7.8 + (-12) + (-82.3)\).
   A. -91.3  
   B. -62.5  
   C. -102.1  
   D. 102.1

16. A soccer team wants new uniforms. A jersey costs $42, shorts cost $25, socks cost $6, and shin guards cost $18. How much does one uniform cost?
   A. $62  
   B. $74  
   C. $83  
   D. $91

17. Tessa had $110 in her checking account. She paid her cable bill for $75. She deposited $50 from her part-time job before writing a check for $85 to pay her credit card bill. What is Tessa’s account balance?
   A. $320  
   B. $170  
   C. $-100  
   D. $0

18. The lowest temperature ever recorded in Minneapolis was \(-41^\circ F\). The lowest temperature ever recorded in Chicago was \(-27^\circ F\). Was the Chicago temperature warmer or colder, and by how many degrees?
   A. 14° warmer  
   B. 68° colder  
   C. 14° colder  
   D. 68° warmer
19. Find $(4)(-1)(-3)(11)(-2)$?
   A. $-264$
   B. $44$
   C. $264$
   D. $132$

20. Which of the following expressions is NOT equivalent to $\frac{80}{10}$?
   A. $\frac{80}{10}$
   B. $\frac{10}{80}$
   C. $8$
   D. $80 \div 10$

21. What is the value of $\frac{70}{0}$?
   A. $7$
   B. $70$
   C. $0$
   D. undefined

22. A wall in Mark’s bedroom is $8\frac{2}{5}$ feet high and $18\frac{1}{3}$ feet long. If he paints $\frac{1}{2}$ of the wall blue, how many square feet will be blue?
   A. $144 \frac{1}{15}$
   B. $72 \frac{1}{15}$
   C. $77$
   D. $154$

23. Match each expression on the left with an equivalent expression on the right.
   A. $(2 \cdot 8) \cdot 7$  
   B. $2 + 7 + 8$
   C. $5 \cdot 3 \cdot 12$
   D. $3 + (5 + 12)$
   E. $(3 + 5) + 12$
   F. $8 + 2 + 7$
   G. $2 \cdot (8 \cdot 7)$
   H. $5 \cdot 12 \cdot 3$

24. Simplify: $|3 - 10| - (12 \div 4 + 2)^2$
   A. $-6$
   B. $3$
   C. $-18$
   D. $-32$

25. Simplify: $\frac{8|7 + (-10)|}{2 \cdot 3^2}$
   A. $-\frac{4}{3}$
   B. $\frac{4}{3}$
   C. $\frac{4}{3}$
   D. $-\frac{2}{3}$
26. Simplify: \((10 - 3)^2 - 6 + 2(8 - \sqrt{64})\)
   A. 49
   B. 43
   C. 7
   D. 19

27. Write 5.36 as a simplified fraction or mixed number as appropriate.
   A. \(5 \frac{9}{250}\)
   B. \(5 \frac{18}{50}\)
   C. \(5 \frac{9}{25}\)
   D. \(\frac{9}{25}\)

28. Which of the following sentences is true?
   A. 8.43 < 8.432
   B. 28.37 < 28.29
   C. 94.745 > 94.752
   D. 7.1 > 7.24

29. Tad wants to attend a sports camp this spring that costs $170.00 for the week. He rakes leaves during the fall and earns $58.35. He shovels snow during the winter and earns $85.80. How much more money does Tad need to earn to pay for the sports camp? Write your answer as a decimal.
   Tad needs to earn $__________ .

30. Sara drives 72 miles on 3.2 gallons of gas. She uses this information to calculate how many miles per gallon she can drive. Using this result, how many miles can Sara drive on 13.3 gallons of gas?
   A. 16.9
   B. 1.7
   C. 299.25
   D. 29.925

31. A 16-ounce bottle of Spring Water is $2.08. A 20-ounce bottle of Fresh Water is $2.40. Which statement about the unit prices is true?
   A. Spring Water has a higher unit price of $0.13 per ounce.
   B. Fresh Water has a higher unit price of $0.12 per ounce.
   C. Fresh Water has a higher unit price of $0.13 per ounce.
   D. Spring Water has a higher unit price of $0.12 per ounce.

32. Pete’s family went out to eat. The bill for the food was $36.25. The restaurant tax was $3.37 and Pete left a tip for $5.75. What is the best estimate for how much the meal cost altogether?
   A. $45.00
   B. $47.00
   C. $40.00
   D. $45.37
33. Jim rode his bike \( \frac{3}{5} \) miles to the train station. He rode the train into downtown. He then rode his bike \( \frac{3}{4} \) miles to get to work. How many miles total does Jim ride his bike on his way to work?
A. \( 14 \frac{7}{9} \)
B. \( 15 \frac{11}{20} \)
C. \( 14 \frac{11}{20} \)
D. \( 15 \frac{1}{2} \)

34. Which choice shows \( 29 + 22 + 11 \) rewritten correctly using the commutative property and then simplified correctly?
A. \( 11 + 9 + 20 + 22 = 20 + 22 = 42 \)
B. \( 29 + 11 + 22 = 40 + 33 = 73 \)
C. \( 29 + 33 = 62 \)
D. \( 29 + 11 + 22 = 40 + 22 = 62 \)

35. Which choice shows \( (40 + 10) + 30 \) rewritten correctly using the associative property and then simplified correctly?
A. \( 40 + (10 + 30) = 50 + 30 = 80 \)
B. \( 40 + 30 + 10 = 70 + 10 = 80 \)
C. \( 40 + (10 + 30) = 40 + 40 = 80 \)
D. \( (10 + 40) + 30 = 50 + 30 = 80 \)

36. Which equation is equivalent to: \( 15 - 7x = 14 \)?
A. \( 8x = 14 \)
B. \( -8x = 14 \)
C. \( 7x = -1 \)
D. \( 7x = 1 \)

37. Which of the following equations has the solution \( x = \text{all real numbers} \)?
A. \( 4(3 - x) + 6x = 3x + 12 + 2x \)
B. \( 4(3 - x) + 6x = 3x + 12 - x \)
C. \( 4(3 - x) + 6x = x + 12 - 3x \)
D. \( 4(3 - x) + 6x = 3x + 10 - x \)

38. Ella rents a car from a company that rents cars by the hour. She has to pay an initial fee of $52, and then they charge her $8 per hour. She has $144 available to spend on car rental. What is the greatest number of hours for which she can rent the car? (The car cannot be rented for part of an hour.)
A. 11
B. 18
C. 12
D. 11.5

39. Solve the formula: \( M = 2P + 3Q \) for the variable \( Q \).
A. \( P = \frac{M - 3Q}{2} \)
B. \( Q = 3(M - 2P) \)
C. \( Q = \frac{M - 2P}{3} \)
40. The solution to which inequality is graphed below?

A. $4s + 4 \geq -28$
B. $-3s - 4 \geq 14$
C. $-2s - 7 \leq 5$
D. $3s + 8 > -10$

41. Match each compound inequality on the left to the graph that represents its solution on the right.

A. $-5x + 9 < -6 \text{ or } -3x + 1 \geq 7$
B. $-6x > -18 \text{ and } 1 \leq 2x + 5$
C. $-16 \leq 6x + 2 < 14$
D. $3m + 18 > 33 \text{ and } -8m \geq 32$
E. $3m \geq -12 \text{ and } -m - 6 < 3$
F. $-2m - 14 \leq -22 \text{ and } 3m + 14 \geq 8$

42. For which of the following compound inequalities is there no solution?

A. $3m + 18 > 33 \text{ and } -8m \geq 32$
B. $3m \geq -12 \text{ and } -m - 6 < 3$
C. $-2m - 14 \leq -22 \text{ and } 3m + 14 \geq 8$
D. $3m > 15 \text{ and } -9m < 18$

43. Which of the following ordered pairs is represented by a point located on the y-axis?

A. (4,1)
B. (-7,0)
C. (-4,-1)
D. (0,5)
44. What is the graph of the equation $x = -4$?

A. 

B. 

C. 

D. 

45. The candy store is open for 14 hours per day. They sell an average of 5 candies per hour. The back storeroom currently has 560 candies in it. Write an equation that estimates the number of candies $T$ that will be in the storeroom after $H$ hours. How many days will it take to run out of candies?

A. $H = 560 - 5T$; The store will run out in 8 days
B. $H = 560 - 5T$; The store will run out in 112 days
C. $T = 560 - 5H$; The store will run out in 112 days
D. $T = 560 - 5H$; The store will run out in 8 days
46. Match the line described on the left with the slope of the line on the right.
   A. \( y = 0 \)  
   B. \( 9x + 3y = 18 \)  
   C. the line through \((-5,7)\) and \((-5,-8)\)  
   D. the line through \((4,-1)\) and \((8,7)\)  
   E. \(-3\)  
   F. \(0\)  
   G. \(2\)  
   H. Undefined

47. Which line has an undefined slope?
   A. \( y = -5 \)  
   B. \( y = x - 5 \)  
   C. \( 3y - 6x = 0 \)  
   D. \( x = 10 \)

48. Which equation describes the line whose \(x\)-intercept is \((4,0)\) and whose \(y\)-intercept is \((0,-3)\)?
   A. \( y = 4x - 3 \)  
   B. \( y = -\frac{4}{3}x + 4 \)  
   C. \( y = -3x + 4 \)  
   D. \( y = \frac{3}{4}x - 3 \)

49. Find the equation of the line that contains the point \((4,-2)\) and is perpendicular to the line \( y = -2x + 8 \)?
   A. \( y = -2x + 6 \)  
   B. \( y = \frac{1}{2}x - 4 \)  
   C. \( y = -\frac{1}{2}x \)  
   D. \( y = 2x - 10 \)

50. Which of the following lines is parallel to the line \( y = -\frac{3}{2}x - 4 \)?
   A. \( y = -\frac{3}{2}x + 5 \)  
   B. \( y = \frac{3}{2}x - 3 \)  
   C. \( y = -\frac{3}{2}x - 4 \)  
   D. \( y = \frac{3}{2}x + 1 \)

51. Multiply and simplify: \( \frac{45a^3}{a^2-4} \cdot \frac{a^2+2a}{30a^2} \)
   A. \( \frac{3a}{a-2} \)  
   B. \( \frac{2a-4}{9a^2} \)  
   C. \( \frac{6a-12}{3a^2} \)  
   D. \( \frac{3a}{-4} \)
52. Divide and simplify: $\frac{5y}{y^2+4y+4} + \frac{20y^2}{y^2-4}$

A. $\frac{1}{4y}$  
B. $\frac{y^2-4}{4y(y^2+4y+4)}$  
C. $\frac{y-2}{4y(y+2)}$  
D. $\frac{y-2}{4y^2+8y}$

53. Subtract and state the sum in simplest form: $\frac{5}{x-3} - \frac{9}{x-1}$

A. $\frac{-4x+22}{x^2-4x+3}$  
B. $\frac{-4x-32}{x^2-4x+3}$  
C. $\frac{-4}{x^2-4x+3}$  
D. $\frac{1}{x^2-4x+3}$

54. Simplify: $\frac{5\sqrt{7}}{4x^2}$

A. $\frac{5x}{7}$  
B. $35x$  
C. $\frac{20x^2-x}{28x-7}$  
D. $\frac{10x}{7}$

55. Perform the indicated operations and simplify: $\frac{1}{6a^2} - \frac{5b}{3a^3} + \frac{b^2}{4a}$

A. $\frac{3a^2b^2+2a-20b}{12a^3}$  
B. $\frac{b^2-5b+1}{7a^6}$  
C. $\frac{1-5b+b^2}{6a^2-3a^3+4a}$  
D. $\frac{b^2-5b+1}{12a^3}$

56. For the following equation, _______ is an excluded value: $\frac{2x}{x+7} + \frac{3x+1}{x+7} = \frac{1}{2}$

A. $-\frac{1}{3}$  
B. 7  
C. $-7$  
D. 0

57. One student can paint a wall in 16 minutes. Another student can paint the same wall in 24 minutes. Working together, about how long will it take for them to paint the wall?

A. 6 minutes  
B. 10 minutes  
C. 14 minutes  
D. 20 minutes
58. Given \( f(x) = 3x - 5 \), find \( f(x - 2) \)
   A. \( f(x - 2) = 3x - 7 \)
   B. \( f(x - 2) = 3x - 11 \)
   C. \( f(x - 2) = x - 7 \)
   D. \( f(x - 2) = 3x^2 - 11x + 10 \)

59. The graph of a function \( f(x) \) passes through the following points: (0,2), (1,0), (-1,0).
   Which of the following could be \( f(x) \)?
   A. \( f(x) = -2x^2 + 2 \)
   B. \( f(x) = 2x + 2 \)
   C. \( f(x) = -2\sqrt{x} + 2 \)
   D. \( f(x) = -2x + 2 \)

60. Graph the function: \( f(x) = 3x^2 - 12x + 6 \)
   A. 
   B. 
   C. 
   D. 

61. Graph the function: \( f(x) = \frac{1}{2} \sqrt{x - 4} + 1 \)

A.

B.

C.

D.

62. What are the domain and range of the real-valued function \( f(x) = -3 + \sqrt{4x - 12} \)?

A. The domain is \( x \geq 3 \), the range is \( f(x) \leq -3 \)
B. The domain is \( x \geq 3 \), the range is all real numbers.
C. The domain is \( x \geq 3 \), the range is \( f(x) \geq -3 \)
D. The domain is \( x \leq 3 \), the range is \( f(x) \geq -3 \)

63. \( f(x) = 2x^3 - 4x + 6 \); \( g(x) = 2x^3 + 3x^2 - 4x + 2 \); Find \( (f - g)(x) \).

A. \( 3x^2 - 8x + 8 \)
B. \( -3x^2 + 4 \)
C. \( 3x^2 - 4 \)
D. \( 4x^3 + 3x^2 - 8x + 8 \)

64. \( f(x) = x^2 + x - 6 \); \( g(x) = x + 3 \); Find \( \left( \frac{f}{g} \right)(x) \).

A. \( x - 2, x \neq -3 \)
B. \( x^2 - 3, x \neq -3 \)
C. \( x - 3, x \neq -3 \)
D. \( x^2 + x - 2, x \neq -3 \)
65. Three polynomials are factored below, but some coefficients and constants are missing.
   All of the missing values of $a$, $b$, $c$, and $d$ are integers.
   A. $x^2 + 2x - 8 = (ax + b)(cx + d)$
   B. $2x^3 + 2x^2 - 24x = 2x(ax + b)(cx + d)$
   C. $6x^2 - 15x - 9 = (ax + b)(cx + d)$

   Fill in the table with the missing values of $a$, $b$, $c$, and $d$

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66. Which factor do $9x^2 - 12x + 4$ and $9x^2 - 4$ have in common?
   A. $3x - 2$
   B. $9x^2$
   C. $3x + 4$
   D. $3x + 2$

67. Find the number to add to $x^2 - 12x$ to make it a perfect square trinomial.
   The solution is __________.

68. Which of the following are perfect square trinomials?
   I. $4x^2 + 9$
   II. $x^2 - 8x + 16$
   A. Only II
   B. Both I and II
   C. Only I
   D. Neither I nor II

69. A rectangular vegetable garden will have a width that is 4 feet less than the length, and an area of 140 square feet. If $x$ represents the length, then the length can be found by solving the equation: $x(x - 4) = 140$. What is the length of the garden?
   The length is __________ feet.

70. Match each expression on the left with an equivalent expression on the right.
   A. $\sqrt{24a^5b^4}$
   B. $\sqrt[3]{-24a^5b^4}$
   C. $-\sqrt{24a^6b^4}$
   D. $2a^2b^2\sqrt{6a}$
   E. $-2a^3b^2\sqrt[6]{6}$
   F. $-2ab\sqrt[3]{3a^2b}$

71. Simplify $\sqrt[3]{27x^7}$
   A. $9x^2\sqrt{x}$
   B. $3x^4$
   C. $3x^3\sqrt{x^2}$
   D. $3x^2\sqrt{x}$
72. Simplify \( \frac{h^3}{h^2} \)
   A. \( h^2 \)
   B. \( h^3 \sqrt{h} \)
   C. \( \sqrt{h} \)
   D. \( \frac{3\sqrt{h}}{h^2} \)

73. Perform the operations and simplify \( \frac{\sqrt{6x^3}}{\sqrt{3x}} \cdot \sqrt{8x^2} \)
   A. \( 4x\sqrt{x^3} \)
   B. \( 4x^2 \)
   C. \( 16x^4 \)
   D. \( \sqrt{16x^4} \)

74. Use the Quadratic Formula to solve the equation: \( 2x^2 + 1 = 8x \)
   A. \( x = 2 + \frac{\sqrt{14}}{2} \) or \( x = 2 - \frac{\sqrt{14}}{2} \)
   B. \( x = -2 + \frac{\sqrt{14}}{2} \) or \( x = -2 - \frac{\sqrt{14}}{2} \)
   C. \( x = 4 + \sqrt{14} \) or \( x = 4 - \sqrt{14} \)
   D. \( x = -\frac{1}{4} + \frac{\sqrt{65}}{4} \) or \( x = -\frac{1}{4} - \frac{\sqrt{65}}{4} \)

75. Solve \( x^2 - 16x = -40 \)
   A. \( -8 \pm \sqrt{24} \)
   B. \( 2\sqrt{6} \pm 8 \)
   C. \( 8 \pm 2\sqrt{6} \)
   D. \( 8 + \sqrt{24} \)

76. Rationalize the denominator and simplify \( \frac{3+\sqrt{x}}{2-\sqrt{x}} \)
   A. \( \frac{6+x}{4-x} \)
   B. \( \frac{11\sqrt{x}+x}{4+x} \)
   C. \( \frac{6-x-5\sqrt{x}}{4+x} \)
   D. \( \frac{6+x+5\sqrt{x}}{4-x} \)

77. The period \( T \) (in seconds) of a pendulum is given by \( T = 2\pi \sqrt{\frac{L}{32}} \) where \( L \) stands for the length (in feet) of the pendulum. If \( \pi = 3.14 \) and the period is 12.56 seconds, what is the length?

The length of the pendulum is _________ feet.
78. A photograph has a length that is 3 inches longer than its width, x. So its area is given by the expression \( x(x + 3) \) square inches. If the area of the photograph is 70 square inches, what is the length of the photograph?

The length of the photograph is \__________\ inches.

79. An arrow is shot with an initial upward velocity of 100 feet per second from a height of 5 feet above the ground. The equation \( h = -16t^2 + 100t + 5 \) models the height in feet \( t \) seconds after the arrow is shot. After the arrow passes its maximum height, it comes down and hits a target that was placed 20 feet above the ground. About how long after the arrow was shot does it hit its intended target? (a simple four-operation calculator will be allowed for this problem)

A. 0.15 seconds  
B. 6.10 seconds  
C. 6.25 seconds  
D. 6.30 seconds

80. Tickets for a classical jazz concert cost $10 for adults and $7 for children. There were 12 tickets sold for a total of $96.00. How many adult tickets were sold?

A. 4  
B. 8  
C. 2  
D. 6
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